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Review for Final Exam

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Outline

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- Hydrodynamic Forces and Moments
- Diffusion Mechanisms
- Particle Adhesion and Detachment
- Particle Charging

Hydrodynamic Forces

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Drag Forces

$$F_D = \frac{3\pi\mu U d}{C_c} C_D$$

$$\mathrm{Re} = \frac{\rho U d}{\mu}$$

$$C_D = \frac{24[1 + 0.15\,\mathrm{Re}^{0.687}]}{\mathrm{Re}}$$

Cunningham Correction

$$C_c = 1 + \frac{2\lambda}{d}[1.257 + 0.4e^{-1.1d/2\lambda}]$$

Lift Force

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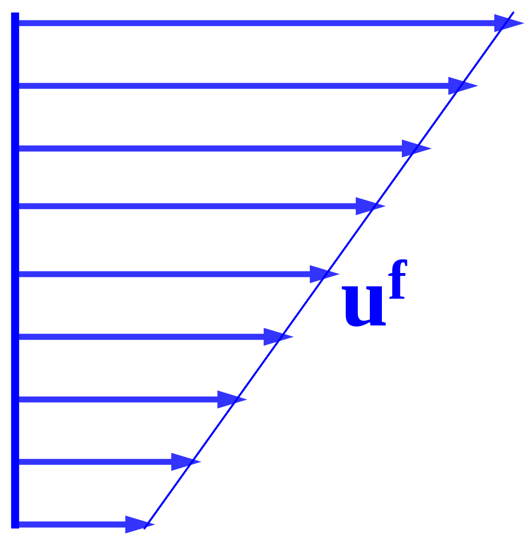

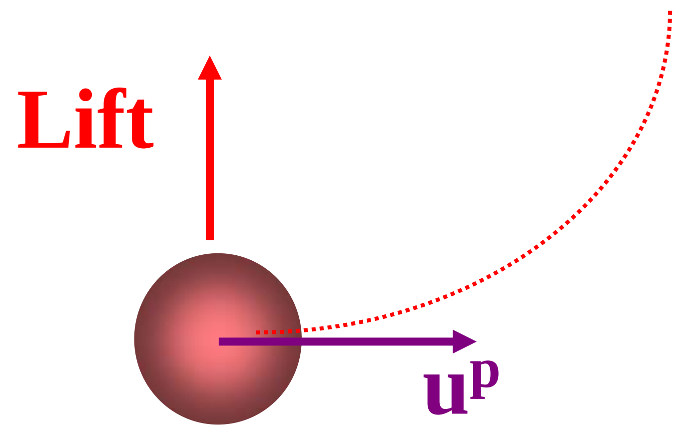

$$\operatorname{sgn}(x) = \begin{cases} +1 & x > 0 \\ -1 & x < 0 \end{cases}$$

Saffman (1965, 1968)

$$F_{L(Saff)} = 1.615 \rho \nu^{1/2} d^2 (u^f - u^p) \left| \frac{du^f}{dy} \right|^{1/2} \operatorname{sgn}\left(\frac{du^f}{dy}\right)$$

Aerosols Particle Motion

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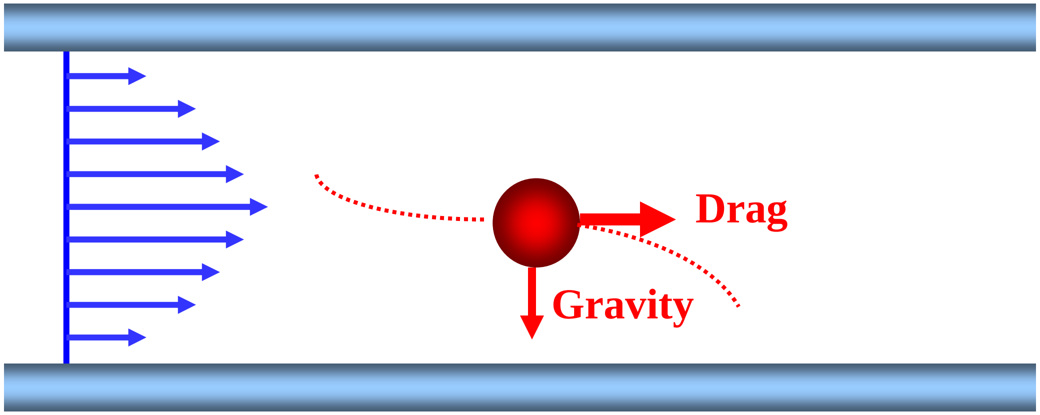

Equation of Motion

$$m\frac{d\mathbf{u}^p}{dt} = \frac{3\pi\mu d}{C_c}(\mathbf{u}^f - \mathbf{u}^p) + m\mathbf{g}$$

Aerosols Particle Motion

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$$\tau \frac{d\mathbf{u}^p}{dt} = (\mathbf{u}^f - \mathbf{u}^p) + \tau \mathbf{g}$$

Relaxation Time

$$\tau = \frac{mC_c}{3\pi\mu d} = \frac{d^2 \rho^p C_c}{18\mu} = \frac{Sd^2 C_c}{18\nu}$$

$$S = \frac{\rho^p}{\rho^f}$$

$$\tau(s) \approx 3 \times 10^{-6} d^2 (\mu m)$$

Viscous Sublayer

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Turbulent stress is negligible

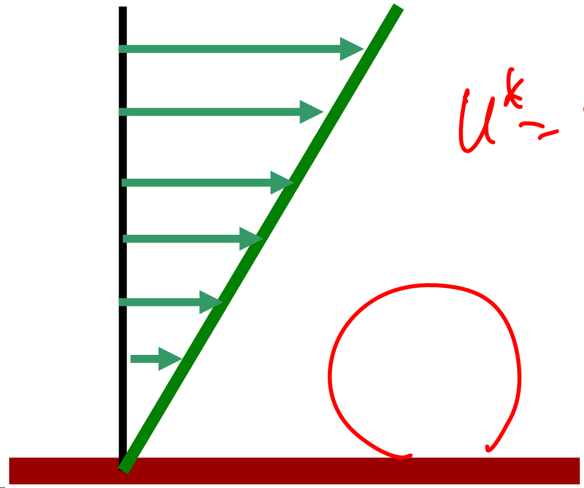
$$\tau_0 = \mu \frac{dU}{dy}$$

$$u^{*2} = \nu \frac{dU}{dy}$$

$$U = \frac{u^{*2} y}{\nu}$$

$$\frac{dU^+}{dy^+} = 1$$

$$u^+ = y^+$$



$u^* \approx \frac{1}{20} U_0$

u^* = shear velocity

$$0 < y^+ \leq 5$$

ν, u^*

$F^+ = \frac{F}{\mu \nu}$

$$F^+_{L(Saff)} = 0.807 d^{+3}$$

$d^+ = \frac{d u^*}{\nu}$

Diffusion and Fick's Law

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Fick's Law

$$J = -D\frac{dc}{dx}$$

Diffusion Equation

$$\frac{\partial c}{\partial t} + \mathbf{v}\cdot\nabla c = D\nabla^2 c$$

Diffusivity

$$D = \frac{kTC_c}{3\pi\mu d}$$

Diffusion

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- Similarity Method
- Separation of Variable Method
- Integral Method

Particle Adhesion and Detachment

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- van der Waals Force
- JKR Adhesion Model
- DMT Adhesion Model
- Maugis-Pollock Model
- Particle Detachment Mechanisms
- Maximum Moment Resistance

JKR Model

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Johnson-Kandall-Roberts (1971)

$$a^3 = \frac{d}{2K}\left[P + \frac{3}{2}W_A \pi d + \sqrt{3\pi W_A dP + \left(\frac{3\pi W_A d}{2}\right)^2}\right]$$

$$K = \frac{4}{3}\left[\frac{1-\upsilon_1^2}{E_1} + \frac{1-\upsilon_2^2}{E_2}\right]^{-1}$$

Hertz Model

$$a^3 = \frac{dP}{2K}$$

DMT Model

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Derjaguin-Muller-Toporov (1975)

Pull-Off Force

$$F_{Po}^{DMT} = \pi W_A d$$

$$F_{Po}^{DMT} = \frac{4}{3} F_{Po}^{JKR}$$

Contact Radius at Zero Force

$$a_0 = \left(\frac{\pi W_A d^2}{2K} \right)^{\frac{1}{3}}$$

Contact Radius at Separation

$$a = 0$$

Maugis-Pollock Model

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$$P + \pi W_A d = \pi a^2 H$$

$$H = 3Y$$

$$a_0 \sim d^{\frac{2}{3}}$$ ↔ **Elastic**

$$a_0 \sim d^{\frac{1}{2}}$$ ↔ **Plastic**

JKR Model

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$$a^{*3} = 1 - P^* + \sqrt{1 - 2P^*}$$

$$P^* = -\frac{P}{\frac{3}{2}\pi W_A d}$$

$$a^* = \frac{a}{\left(\frac{3\pi W_A d^2}{4K}\right)^{\frac{1}{3}}}$$

$$M^{*JKR} = P^* a^* = P^* (1 - P^* + \sqrt{1 - 2P^*})^{1/3}$$

$$M^{*JKR}_{max} = 0.42$$

Aerosols Charging and Their Kinetics

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Coulomb Force

$$\mathbf{F}_E = q\mathbf{E}$$

$$q = ne$$

Particle Mobility

$$u = Z^p = \frac{qC_c}{3\pi\mu d}$$

Particle Charging

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Boltzmann Equilibrium Charge Distribution

$$f(n) = \frac{0.24}{\sqrt{d\pi}} \exp\{-\frac{0.05n^2}{d}\}$$

$$d > 0.02\mu m$$

$$\bar{n} = \approx 2.36\sqrt{d}, \quad d(\mu m)$$

Diffusion Charging

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$$n = \frac{dkT}{2e^2} \ln[1 + (\frac{2\pi}{m_i kT})^{1/2} n_{i\infty} d e^2 t]$$

Field Charging

$$n_\infty = [1 + \frac{2(\varepsilon_p - 1)}{\varepsilon_p + 2}] \frac{Ed^2}{4e} \text{ as } t \to \infty$$