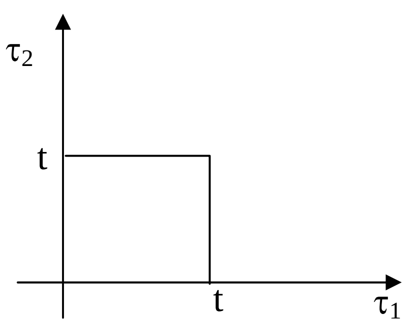
ME437/ME537 **Hints for HW SET 2**

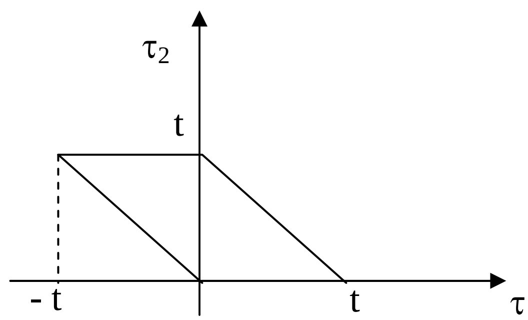
1. For a particle of mass m in an air stream with a constant velocity $\mathbf{u}^f$ and in gravitational field, evaluate the velocity and position vectors of the particle as a function of time. Also evaluate the terminal velocity of the particle when a constant velocity $\mathbf{u}^f$ is present.

   Start with the governing equations. Solve the first order differential equation with constant coefficient. The answer is in the note.

2. Show that the mean square response of particle position given by $\overline{x^2(t)} = \int_0^t \int_0^t R_{uu}(\tau_1 - \tau_2) d\tau_1 d\tau_2$ may be restated as $\overline{x^2(t)} = 2\int_0^t (t-\tau) R_{uu}(\tau) d\tau$, and the diffusivity is given by $D = \int_0^\infty R_{uu}(\tau) d\tau$.

   Change variables of integration form τ_1 and τ_2 to τ_2 and $\tau = \tau_1 - \tau_2$. Draw a picture of the domain of integration in τ_1, τ_2 and in τ, τ_2 and identify the limits of integral in the new domain.





3. Evaluate the concentration of uniform size spheres with a constant terminal velocity v^t. Assume that $c = c_o$ at $x = x_o$, and $c = c(x)$ with x being in the vertical direction. Note that the generalized mass diffusion equation is given as $\frac{\partial c}{\partial t} + (\mathbf{v} + v^t) \cdot \nabla c = D \nabla^2 c$

   Diffusion equation becomes

$$-v^t \frac{dc}{dx} = D \frac{d^2 c}{dx^2}$$

4. Consider the case of diffusion to a wall governed by $\frac{\partial c}{\partial t} = D\frac{\partial^2 c}{\partial y^2}$, $c(0,t) = 0$, $c(\infty,t) = c_o$ and $c(y,0) = c_o$. Use the integral method with an approximate expression for the profile given by $\frac{c}{c_o} = 2\frac{y}{\delta_c} - (\frac{y}{\delta_c})^2$ and evaluate the variation of diffusion boundary layer thickness δ_c with time.

   Integrate the diffusion equation from 0 to δ_c.

$$\int_0^{\delta_c} (\frac{\partial c}{\partial t} = D\frac{\partial^2 c}{\partial y^2})dy \quad \text{or} \quad \frac{\partial}{\partial t}\int_0^{\delta_c} cdy - c_o\frac{d\delta_c}{dy} = -D\frac{\partial c}{\partial y}\Big|_{y=0}$$

   Now use the suggested profile and evaluate the terms needed in the equations.

   Answer $\delta_c = \sqrt{12Dt}$

5. Evaluate the variation of concentration with space and time in a region between two parallel plates with an initially uniform concentration. Note that $\frac{\partial c}{\partial t} = D\frac{\partial^2 c}{\partial y^2}$, $c(0,t) = 0$, $c(b,t) = 0$ and $c(y,0) = c_o$. [Hint: use the method of separation of variables.]

6. The diffusion equation in cylindrical coordinate is given as $\frac{\partial c}{\partial t} = \frac{D}{r}\frac{\partial}{\partial r}(r\frac{\partial c}{\partial r})$. Reduce the diffusion equation to a similarity form by assuming that $c(r,t) = \frac{1}{r}z(\eta)$, where $\eta = \frac{r}{\sqrt{4Dt}}$.

7. Develop a sample white noise for Brownian excitation acting on 0.05 μm particles in air at room temperature. Evaluate a sample particle trajectory when the there is a uniform air flow velocity of 0.1 m/s in x direction.