ME 637 - Particle-II

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PHENOMENOLOGICAL MODELS

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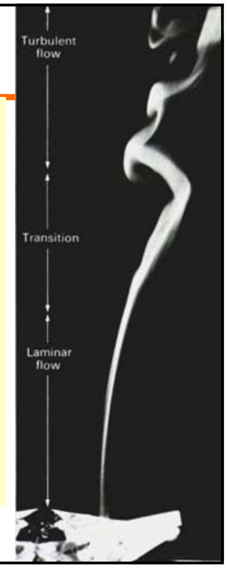
Phenomenological Models

Outline

- Reynolds Equation
- Eddy Viscosity Models
- Mixing Length Model
- Near Wall Flows



Phenomenological Models For Turbulence

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Navier-Stokes

$$\rho\left(\frac{\partial u_i}{\partial t}+u_j\frac{\partial u_i}{\partial x_j}\right)=-\frac{\partial p}{\partial x_i}+\mu\frac{\partial^2 u_i}{\partial x_j \partial x_j}$$

$$\frac{\partial u_i}{\partial x_i}=0$$

Turbulence

$$\mathbf{u}=\mathbf{U}+\mathbf{u}' \qquad U_i=\overline{u_i} \qquad \overline{u_i'}=0$$

$$p=P+p' \qquad P=\overline{p} \qquad \overline{p'}=0$$

Features of Turbulence

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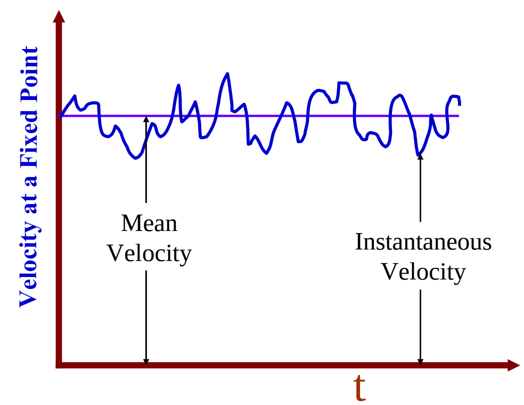

Averaging

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Time Averaging ⇒ $\overline{u_i} = \lim_{T\to\infty} \frac{1}{T}\int_{t_0}^{t+T} u_i dt$

Ensemble Averaging ⇒ $< u_i >= \int_{-\infty}^{+\infty} u_i f(\mathbf{u}) d\mathbf{u}$

Ergodicity ⇒ $\overline{u_i} =< u_i >= U_i$

Averaging

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Properties

$\overline{u_i'} = 0$ $\quad \overline{p'} = 0$

$\overline{u_i' u_j'} \neq 0$ $\quad \overline{p' u_i'} \neq 0$ $\quad \overline{u_i' u_j' u_k'} \neq 0$

$\overline{U_i u_j'} = U_i \overline{u_j'} = 0$ $\quad \frac{\partial \overline{u_i'}}{\partial x_j} = 0$

Phenomenological Models For Turbulence

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Reynolds Equation

$$\rho\left(\frac{\partial U_i}{\partial t} + U_j \frac{\partial U_i}{\partial x_j}\right) = -\frac{\partial P}{\partial x_i} + \mu \frac{\partial^2 U_i}{\partial x_j \partial x_j} - \rho \frac{\partial \overline{u_i' u_j'}}{\partial x_j}$$

$$\frac{\partial U_i}{\partial x_i} = 0$$

Turbulence Stress ⇒ $\tau_{ij}^T = -\rho \overline{u_i' u_j'} = \tau_{ji}^T$

Phenomenological Models For Turbulence

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Reynolds Equation

$$\rho\left(\frac{\partial U_i}{\partial t} + U_j \frac{\partial U_i}{\partial x_j}\right) = \frac{\partial}{\partial x_j}\left[-P\delta_{ij} + \mu\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right) - \rho \overline{u_i' u_j'}\right]$$

Turbulence Stress

Reynolds Stress

$$\boldsymbol{\tau}^T = \begin{pmatrix} -\rho\overline{u'^2} & -\rho\overline{u'v'} & -\rho\overline{u'w'} \\ -\rho\overline{u'v'} & -\rho\overline{v'^2} & -\rho\overline{v'w'} \\ -\rho\overline{u'w'} & -\rho\overline{v'w'} & -\rho\overline{w'^2} \end{pmatrix}$$

Phenomenological Models For Turbulence

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Boussineq Eddy Viscosity Model

$$\tau_{ij}^{T} = -\frac{\rho \overline{u'_k u'_k}}{3}\delta_{ij} + \mu_T\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right)$$

Eddy Viscosity → $\mu_T = \rho \nu_T$

$$\tau_{12}^{T} = \tau^{T} = \rho \nu_T \frac{\partial U}{\partial y}$$

Prandtl Mixing Length Model

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Prandtl Assumption

Ludwig Prandtl

$$(\overline{u'^2})^{\frac{1}{2}} \sim (\overline{v'^2})^{\frac{1}{2}} \sim \ell \frac{dU}{dy}$$

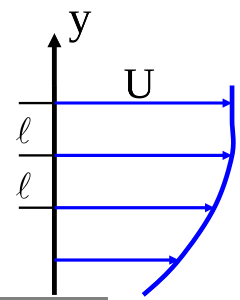
$$\tau^{T} = -\rho \overline{u'v'}$$

Mixing Length

$$\tau^{T} = \rho \ell^2 \left|\frac{\partial U}{\partial y}\right| \frac{\partial U}{dy}$$

Eddy Viscosity

$$\upsilon_T = \ell^2 \left|\frac{\partial U}{\partial y}\right|$$



$$\ell = \kappa \left|\frac{\frac{dU}{dy}}{\frac{d^2U}{dy^2}}\right|$$

von Karman

Velocity Near a Wall

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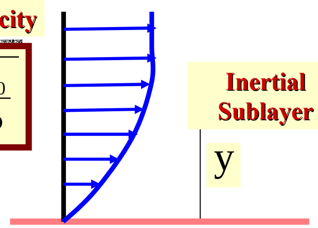
Inertial Sublayer

Turbulence Scales

Shear Velocity

$$u^* = \sqrt{\frac{\tau_0}{\rho}}$$



$$\ell = \kappa y$$

$$\kappa = 0.4$$ ← **von Karman constant**

von Karman

Inertial Sublayer

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Turbulent Stress=Wall Shear Stress

$$\tau_0 = \rho \kappa^2 y^2 \left(\frac{\partial U}{\partial y}\right)^2 \Rightarrow \frac{dU}{dy} = \frac{u^*}{\kappa y}$$

$$\frac{U}{u^*} = U^+ = \frac{1}{\kappa}\ln y + c$$

Wall Units

$$U^+ = \frac{1}{\kappa}\ln y^+ + B$$

$$y^+ = \frac{u^* y}{\nu}$$

$$B \approx 5$$

$$30 < y^+ \le 300$$

Viscous Sublayer

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Turbulent stress is negligible

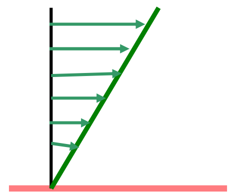
$0 < y^+ \le 5$

$$\tau_0 = \mu \frac{dU}{dy}$$

$$u^{*2} = \nu \frac{dU}{dy}$$

$$\frac{dU^+}{dy^+} = 1$$

$$U^+ = y^+$$



Velocity Near a Wall

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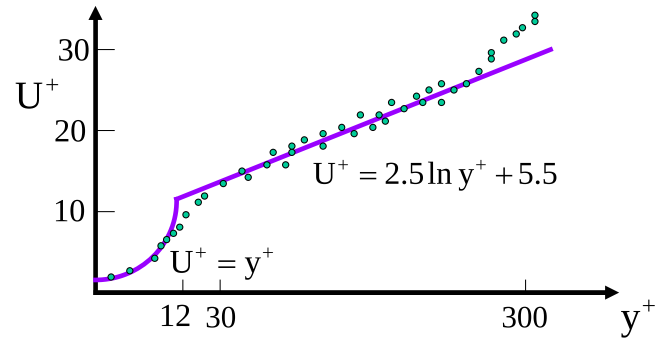

Velocity Near a Wall

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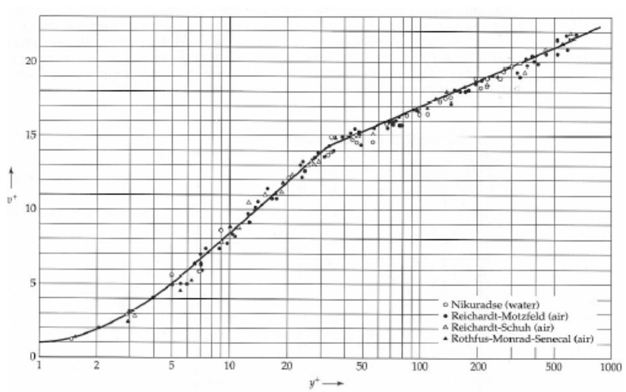

Concluding Remarks

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- ❑ **Reynolds Equation**
- ❑ **Eddy Viscosity Models**
- ❑ **Mixing Length Model**
- ❑ **Near Wall Flows**