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Diffusion to a Cylinder

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Outline

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- Diffusion to a Cylinder in Cross Flow
- Deposition Velocity
- Interception
- Filtration

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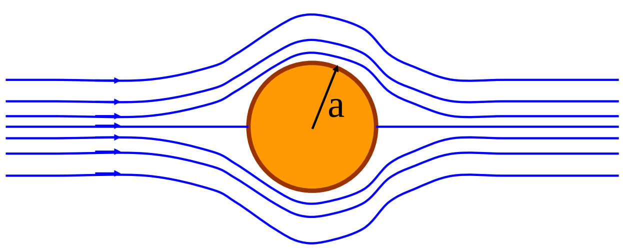

Stream Function

$$\psi = AUa\sin\theta\left[\frac{r}{a}\left(2\ln\frac{r}{a}-1\right)+\frac{a}{r}\right]$$

$$A = \frac{1}{2(2-\ln R_e)}$$

$$Re << 1$$

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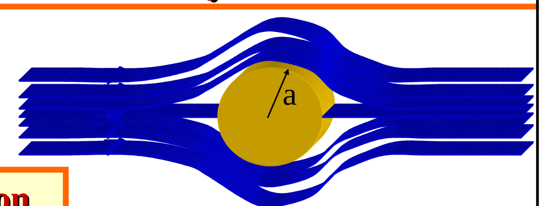

Diffusion Equation

$$\frac{v_\theta}{r}\frac{\partial c}{\partial \theta} + v_r\frac{\partial c}{\partial r} = D\left(\frac{\partial^2 c}{\partial r^2} + \frac{1}{r}\frac{\partial c}{\partial r}\right)$$

Boundary Conditions

$$r = a + \frac{d}{2}, \qquad c = 0$$

$$r = \infty, \qquad c = c_\infty$$

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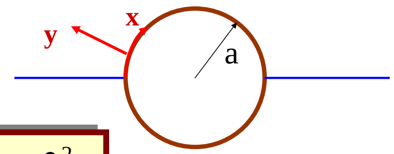
Diffusion Equation



$$u\frac{\partial c}{\partial x}+v\frac{\partial c}{\partial y}=D\frac{\partial^2 c}{\partial y^2}$$

Boundary Conditions

$$y=0, \quad c=0$$
$$y=\infty, \quad c=c_\infty$$

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Stream Function

$$u=\frac{\partial \psi}{\partial y}, \quad v=-\frac{\partial \psi}{\partial x}$$

Using x and ψ

$$\frac{\partial c}{\partial x}=D\frac{\partial}{\partial \psi}[u\frac{\partial c}{\partial \psi}]$$

$$\psi \approx 2AaUy_1^2 \sin x_1$$

$$y_1=\frac{y}{a}, \quad x_1=\frac{x}{a}.$$

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Let

$$\chi=\int \sin^{1/2} x_1 dx_1, \quad \psi_1=\frac{\psi}{2AaU}$$

Diffusion Equation

$$\frac{\partial c}{\partial \chi}=\frac{D}{aAU}\frac{\partial}{\partial \psi_1}(\psi_1^{1/2}\frac{\partial c}{\partial \psi_1})$$

$$\psi_1=0, \quad c=0$$
$$\psi_1=\infty, \quad c=c_\infty$$

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Similarity Equation

$$\xi=\frac{\psi_1}{\chi^{2/3}}$$

$$-\frac{AP_e}{3}\xi\frac{dc}{d\xi}=\frac{d}{d\xi}(\xi^{1/2}\frac{dc}{d\xi})$$

$$c=\frac{c_\infty (AP_e)^{1/3}}{1.45}\int_0^{\sqrt{\xi}} \exp\{-\frac{2}{9}AP_e z^3\}dz$$

$$P_e=\frac{2Ua}{D}=R_e \cdot S_c$$

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Sherwood Number

$$\overline{sh} = \frac{\overline{h}(2a)}{D} = 1.17(AP_e)^{1/3}$$

Collection Efficiency

$$\eta_R = \frac{\overline{h}\pi(2a)c_\infty}{(2a)Uc_\infty} = 3.68A^{1/3}P_e^{-2/3}$$

$$\eta_R \sim d^{-2/3}$$

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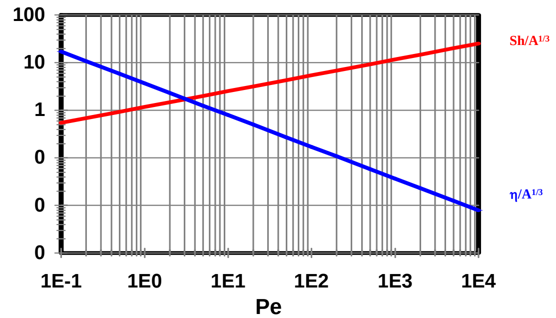

Variations of Sherwood number and collection efficiency with Peclet number .

Direct Interception Limit

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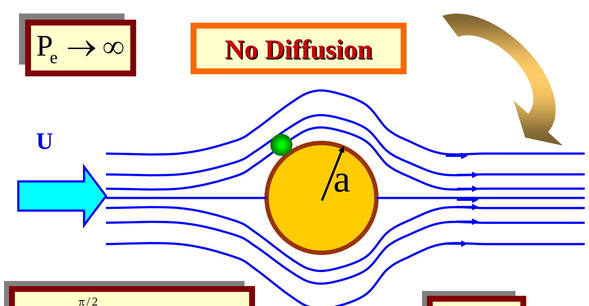
$P_e \to \infty$

No Diffusion



$$\eta_R = -\frac{\int_0^{\pi/2} v\,|_{y=d/2}\,dx}{Ua} = 2AR^2$$

$$R = \frac{d}{2a}$$

Fiber Efficiency

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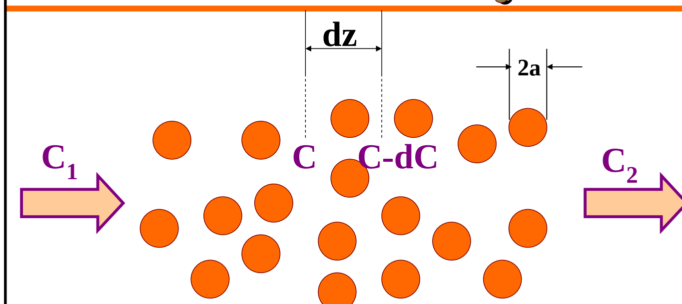


$$dc = -\underbrace{[\eta_R(2a)c]}_{\text{Collection by one fiber}}\underbrace{\left(\frac{\upsilon dz}{\pi a^2}\right)}_{\text{No. of fibers}}$$

$$\eta_R = \frac{\pi a}{2UL}\ln\frac{c_1}{c_2}$$

Fiber Efficiency

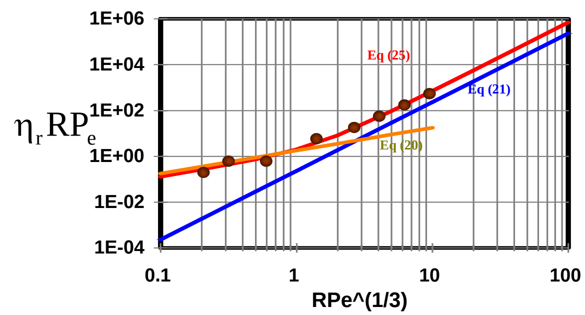
Empirical Equation

$$\eta_r(RP_e) = 1.3RP_e^{1/3} + 0.7(RP_e^{1/3})^3$$

$P_e \to \infty$ $\quad \eta_R \propto R^2$

$P_e \to 0$ $\quad \eta_R \propto P_e^{-2/3}$

Fiber Efficiency



Variation of filter collection efficiency.

Conclusions

- **Deposition by Diffusion to a Cylinder**
- **Deposition by Interception to a Cylinder**
- **Fiber Filter Efficiency**

Thank you!

Questions?