Particle Transport, Deposition and Removal

INTRODUCTION TO AEROSOLS

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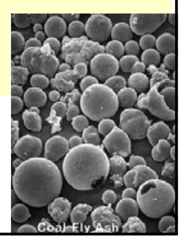
Outline

- Introduction to Aerosols
- Drag Forces
- Cunningham Corrections
- Lift Forces
- Brownian Motion
- Particle Deposition Mechanisms
- Gravitational Sedimentation
- Aerosol Coagulation

Definition

- Aerosols are suspension of solid or liquid particles in a gas.
- Dust, smoke, mists, fog, haze, and smog are common aerosols.
- Aerosol particles are found in different shapes.



Equivalent Diameters

- Equivalent area diameters
- Feret's diameter (maximum distance
- edge to edge)
- Stokes' diameter (diameter of a sphere with the same density and the same velocity as the particle)
- Aerodynamic diameter (diameter of a sphere with the density of water and the same velocity as the particle)

Aerosols in the Atmosphere

	Aerosols	Air
Number Density (Number/cm)	100-10^5	10^{19}
Mean Temperature (K)	240 – 310	240 – 310
Mean Free Path	Greater than 1 m	0.06 μm
Particle Radius	0.01 – 10 μm	2◈ 10^{-4} μm
Particle Mass (g)	10^{-18} - 10^{-9}	4.6 ◈ 10^{-23}
Particle Charge (Elementary Charge Units)	0 – 100	Weakly Ionized Single Charge

Dimensionless Groups

Knudsen Number	$Kn = \frac{2\lambda}{d}$
Mach Number	$M = \frac{\lvert \mathbf{v}^p - \mathbf{v}^f \rvert}{c^f}$
Schmidt Number	$Sc = \frac{\nu}{D} = \frac{n^f \lambda d^2}{4}$
Brown Number	$Br = \left(\frac{\overline{v^{p,2}}}{\overline{v^{f,2}}}\right)^{1/2} = \frac{\lvert \overline{v'^p} \rvert}{\lvert \overline{v'^f} \rvert}$
Reynolds Number	$Re = \frac{\lvert \mathbf{v}^p - \mathbf{v}^f \rvert d}{\nu} = \frac{4M}{K_n}$

Relevant Parameters

λ = Mean Free Path

d = Particle Diameter

v = Particle Velocity

v = Fluid (Air) Velocity

c = Speed of Sound

ν = Kinematic Viscosity

D = Diffusivity

v' = Thermal Velocity

n = Number Density

Mean Free Path

$$\lambda = \frac{1}{\sqrt{2}\pi n d_m^2} = \frac{kT}{\sqrt{2}\pi d_m^2 P}$$

$$k = 1.38 \times 10^{-23}\ J/K$$

Molecular Diameter

Air

$$\lambda(\mu m) = \frac{23.1T}{P}$$

Aerosols Characteristics

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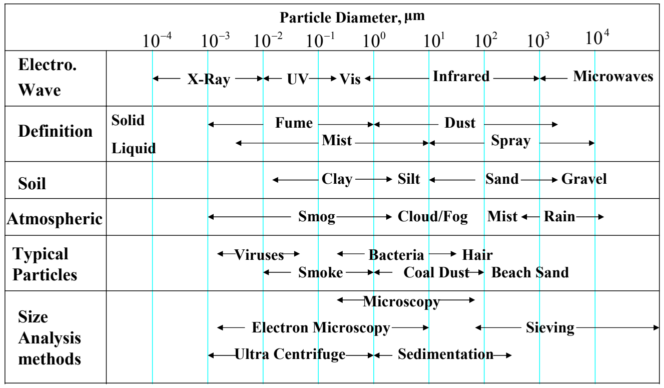

Aerosols Characteristics

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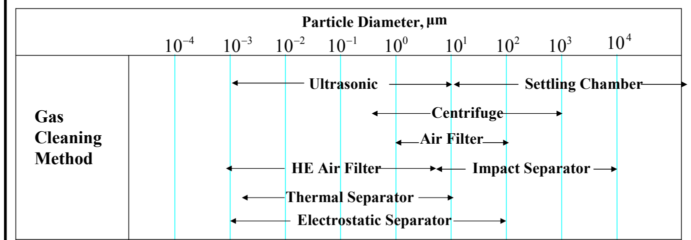

		10^{-3}	10^{-1}	10^{1}	10^{3}
Diffusion Coeff. cm²/s	Air	5×10^{-2}	10^{-5}	2×10^{-9}	2×10^{-11}
	Water	5×10^{-6}	5×10^{-8}	5×10^{-10}	5×10^{-12}
Terminal Velocity cm/s S=2	Air	10^{-6}	2×10^{-4}	0.6	600
	Water	10^{-10}	6×10^{-7}	6×10^{-3}	12

Hydrodynamic Forces

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Drag Forces

Stokes ⇒ $$F = 3\pi\mu U d$$

Drag Coefficient ⇒ $$C_D = \frac{F_D}{\frac{1}{2}\rho U^2 A} = \frac{24}{Re}$$

Reynolds Number ⇒ $$Re = \frac{\rho U d}{\mu}$$

Drag Force for a Sphere

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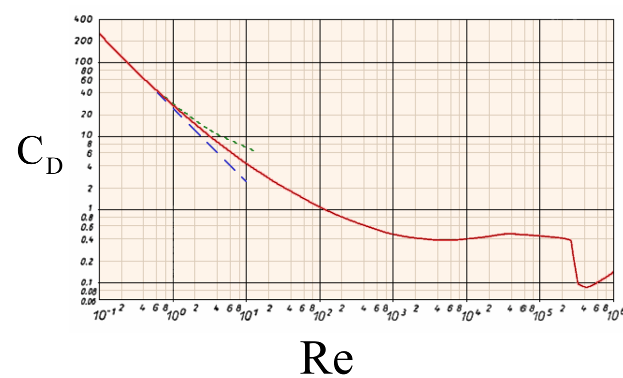

Hydrodynamic Forces

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Drag Forces

Oseen

$$C_D = \frac{24[1 + 3\,Re/16]}{Re}$$

$1 < Re < 1000$

$$C_D = \frac{24[1 + 0.15\,Re^{0.687}]}{Re}$$

Newton

$10^3 < Re < 2.5 \times 10^5$

$$C_D = 0.4$$

Wake Behind a Sphere

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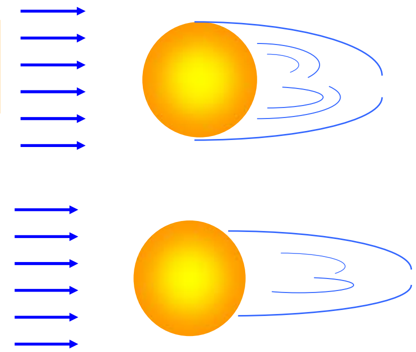
Laminar Boundary Layer

Turbulent Boundary Layer



Drag Force for a Sphere

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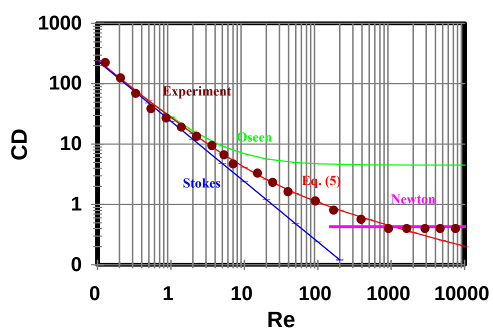

Predictions of various models for drag coefficient for a spherical particle.

Wall Effects

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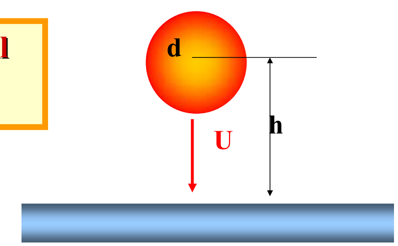
Normal to the Wall

(Bernner, 1961)



$$C_D = \frac{24}{Re}\left(1 + \frac{d}{2h}\right)$$

Wall Effects

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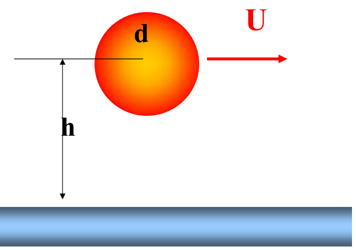
Normal to the Wall (Faxon, 1923)



$$C_D = \frac{24}{Re}\left[1 - \frac{9}{16}\left(\frac{d}{2h}\right) + \frac{1}{8}\left(\frac{d}{2h}\right)^3 - \frac{45}{256}\left(\frac{d}{2h}\right)^4 - \frac{1}{16}\left(\frac{d}{2h}\right)^5\right]^{-1}$$

Cunningham Correction

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For 1000 > Kn > 0

Stokes-Cunningham Drag

$$F_D = \frac{3\pi\mu U d}{C_c}$$

Cunningham Correction

$$C_c = 1 + \frac{2\lambda}{d}[1.257 + 0.4e^{-1.1d/2\lambda}]$$

Cunningham Correction

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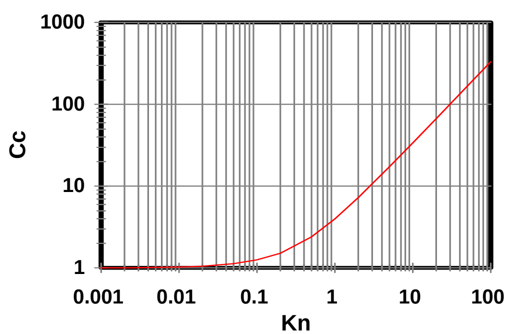

Variation of Cunningham correction with Knudsen number.

Cunningham Correction

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Variations of C_c with d for $\lambda = 0.07$ µm

Diameter, µm	C
10 µm	1.018
1 µm	1.176
0.1 µm	3.015
0.01 µm	23.775
0.001 µm	232.54

Compressibility Effects

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Henderson (1976) **M < 1**

$$C_D = 24\left[\mathrm{Re} + S\left\{4.33 + 1.567\times\exp\left(-0.247\frac{\mathrm{Re}}{S}\right)\right\}\right]^{-1}$$
$$+\exp\left(-\frac{0.5M}{\sqrt{\mathrm{Re}}}\right)\left[\frac{4.5+0.38\left(0.03\mathrm{Re}+0.48\sqrt{\mathrm{Re}}\right)}{1+0.03\mathrm{Re}+0.48\sqrt{\mathrm{Re}}}+0.1M^2+0.2M^8\right]+$$
$$+\left[1-\exp\left(-\frac{M}{\mathrm{Re}}\right)\right]0.6S$$

Compressibility Effects

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Henderson (1976) **M > 1**

$$C_D = \frac{0.9 + \dfrac{0.34}{M^2} + 1.86\left(\dfrac{M}{\mathrm{Re}}\right)^{1/2}\left[2 + \dfrac{2}{S^2} + \dfrac{1.058}{S} - \dfrac{1}{S^4}\right]}{1 + 1.86\left(\dfrac{M}{\mathrm{Re}}\right)^{1/2}}$$

Compressibility Effects

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Carlson and Hoglund (1964)

$$C_D = \frac{24}{\mathrm{Re}}\,\frac{1+\exp\left(-\dfrac{0.427}{M^{4.63}} - \dfrac{3}{\mathrm{Re}^{0.88}}\right)}{1+\dfrac{M}{\mathrm{Re}}\left\{3.82+1.28\exp\left(-1.25\dfrac{\mathrm{Re}}{M}\right)\right\}}$$

Droplets

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$$F_D = 3\pi\mu^f U d\,\frac{1 + 2\mu^f/3\mu^p}{1+\mu^f/\mu^p}$$

For Bubbles → $F_D = 2\pi\mu^f U d$

Non-Spherical Particles

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$$F_D = 3\pi\mu U d_e K$$

$$d_e = \left(\frac{6}{\pi}\text{Volume}\right)^{1/3}$$

K=Correction Factor

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Correction Factor

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Cluster Shape	Correction	Cluster Shape	Correction	Cluster Shape	Correction
oo	K = 1.12	oooo	K = 1.32	oo oo	K = 1.17
ooo	K = 1.27	ooooo	K = 1.45	o o o o o	K = 1.19
o o o	K = 1.16	oooooo	K = 1.57	oo oo oo	K = 1.17
oooooo o o	K = 1.64	ooooooo	K = 1.73		

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Ellipsoidal Particles

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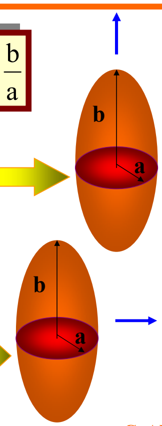
$$F_D = 6\pi\mu U a K'$$

$$\beta = \frac{b}{a}$$

$$K' = \frac{\frac{4}{3}(\beta^2 - 1)}{\frac{(2\beta^2 - 1)}{(\beta^2 - 1)^{1/2}} \ln[\beta + (\beta^2 - 1)^{1/2}] - \beta}$$

$$K' = \frac{\frac{8}{3}(\beta^2 - 1)}{\frac{(2\beta^2 - 3)}{(\beta^2 - 1)^{1/2}} \ln[\beta + (\beta^2 - 1)^{1/2}] + \beta}$$



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Ellipsoidal Particles

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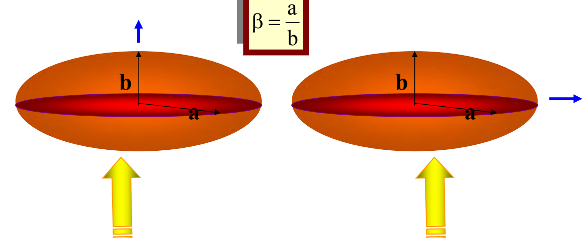
$$\beta = \frac{a}{b}$$



$$K' = \frac{\frac{4}{3}(\beta^2 - 1)}{\frac{\beta(\beta^2 - 2)}{(\beta^2 - 1)^{1/2}} \tan^{-1}(\beta^2 - 1)^{1/2}] + \beta}$$

$$K' = \frac{\frac{8}{3}(\beta^2 - 1)}{\frac{\beta(3\beta^2 - 2)}{(\beta^2 - 1)^{1/2}} \tan^{-1}(\beta^2 - 1)^{1/2}] - \beta}$$

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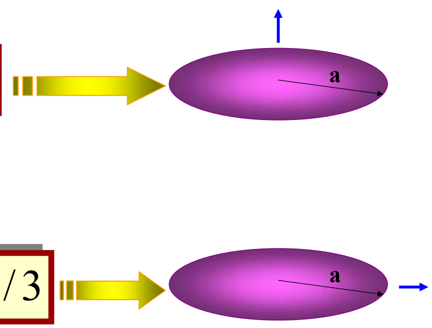
Thin Disks

$$F_D = 16\mu a U$$

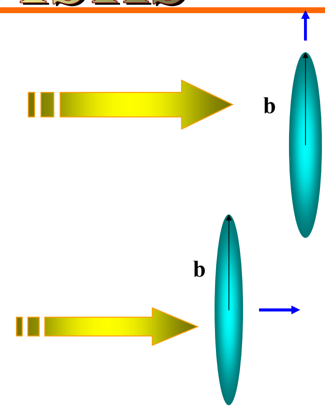


$$F_D = 32\mu a U / 3$$

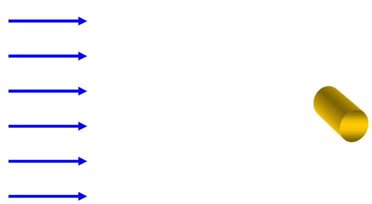
Thin Disks

$$F_D = \frac{4\pi\mu U b}{\ln 2\beta}$$

$$F_D = \frac{8\pi\mu U b}{\ln 2\beta}$$



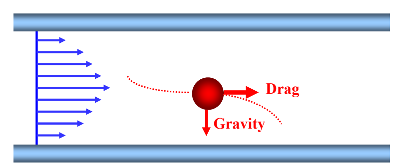
Cylindrical Needle



$$F_D = \frac{4\pi\mu U}{(2.002 - \ln R_e)}$$

$$R_e = \frac{2aU}{\nu}$$

Aerosols Particle Motion



Equation of Motion

$$m\frac{d\mathbf{u}^p}{dt} = \frac{3\pi\mu d}{C_c}(\mathbf{u}^f - \mathbf{u}^p) + m\mathbf{g}$$

Aerosols Particle Motion

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$$\tau \frac{d\mathbf{u}^p}{dt} = (\mathbf{u}^f - \mathbf{u}^p) + \tau \mathbf{g}$$

Relaxation Time

$$\tau = \frac{mC_c}{3\pi\mu d} = \frac{d^2\rho^p C_c}{18\mu} = \frac{Sd^2C_c}{18\nu}$$

$$S = \frac{\rho^p}{\rho^f}$$

$$\tau(s) \approx 3\times10^{-6} d^2(\mu m)$$

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Terminal Velocity

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$$\mathbf{u}^p = (\mathbf{u}^f + \tau\mathbf{g})(1 - e^{-t/\tau})$$

Terminal Velocity = Equilibrium Velocity after Large Time

$$u^t = \tau g = \frac{\rho^p d^2 g C_c}{18\mu}$$

$$u^t(\mu m/s) \approx 30\, d^2(\mu m)$$

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Stopping Distance

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Stopping Distance = Penetration distance for an initial velocity of u_o

$$\mathbf{u}^p = \mathbf{u}_o e^{-t/\tau}$$

$$\mathbf{x}^p = \mathbf{u}_o^p \tau (1 - e^{-t/\tau})$$

$$x^p = u_o^p \tau$$

$$x^p(\mu m) \approx 3\, d^2(\mu m)$$

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Relaxation Time, Terminal Velocity and Stopping Distance

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Diameter, µm	Terminal Velocity	τ sec	Stopping Distance u= 1 m/s	Stopping Distance u= 10 m/s
0.05	0.39 µm/s	4×10^{-8}	0.04 µm	0.0004 mm
0.1	0.93 µm/s	9.1×10^{-8}	0.092 µm	0.0009 mm
0.5	10.1 µm/s	1×10^{-6}	1.03 µm	0.0103 mm
1	35 µm/s	3.6×10^{-6}	3.6 µm	0.0357 mm
5	0.77 mm/s	7.9×10^{-5}	78.6 µm	0.786 mm
10	3.03 mm/s	3.1×10^{-4}	309 µm	3.09 mm
50	7.47 cm/s	7.6×10^{-3}	7.62 mm	76.2 mm

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Particle Path

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$$\mathbf{x}^p = \mathbf{x}^p_o + \mathbf{u}^p_o \tau (1 - e^{-t/\tau})$$
$$+ (\mathbf{u}^f + \tau \mathbf{g})[t - \tau(1 - e^{-t/\tau})]$$

Components

$$x^p / u^f \tau = [t/\tau - (1 - e^{-t/\tau})]$$

$$y^p / u^f \tau = -\alpha[t/\tau - (1 - e^{-t/\tau})]$$

$$\alpha = \frac{\tau g}{u^f \tau}$$

Particle Path

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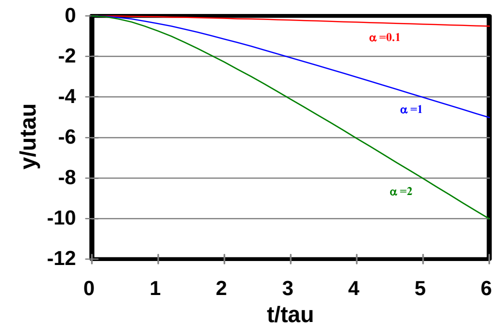

Variations of the particle vertical position with time.

Particle Path

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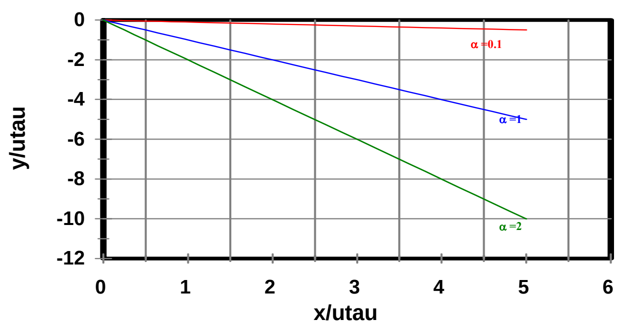

Sample particle trajectories.

Buoyancy Effects

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$$(m + m^a)\frac{d\mathbf{u}^p}{dt} = \frac{3\pi\mu d}{C_c}(\mathbf{u}^f - \mathbf{u}^p) + (m - m^f)\mathbf{g}$$

Fluid Mass ⇒ $m^f = \frac{\pi d^3 \rho^f}{6}$

Apparent Mass ⇒ $m^a = \frac{1}{2} m^f.$

Buoyancy Effects

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$$\left(1+\frac{1}{2S}\right)\tau\frac{d\mathbf{u}^p}{dt} = (\mathbf{u}^f - \mathbf{u}^p) + \tau\mathbf{g}\left(1-\frac{1}{S}\right)$$

Terminal Velocity

$$u^t = \tau g\left(1-\frac{1}{S}\right) = \frac{\rho^p d^2 g C_c}{18\mu}\left(1-\frac{\rho^f}{\rho^p}\right)$$

Lift Force

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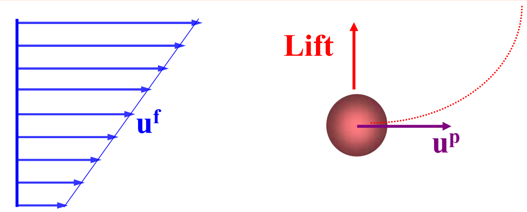

Saffman (1965, 1968)

$$F_{L(Saff)} = 1.615\rho\nu^{1/2}d^2(u^f - u^p)\left|\frac{du^f}{dy}\right|^{1/2}\operatorname{sgn}\left(\frac{du^f}{dy}\right)$$

Saffman Lift Force Constraints

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$$R_{es} = \frac{|u^f - u^p| d}{\nu} << 1 \qquad R_{e\Omega} = \frac{\Omega d^2}{\nu} << 1$$

$$R_{eG} = \frac{\dot{\gamma} d^2}{\nu} << 1 \qquad \varepsilon = \frac{R_{eG}^{1/2}}{R_{es}} >> 1$$

McLaughlin (1991)

Lift Force

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Mai (1992)

$$\frac{F_L}{F_{L(Saff)}} = \begin{cases} (1-0.3314\alpha^{1/2})\exp(-R_{es}/10) + 0.3314\alpha^{1/2} & \text{for } R_{es} \le 40 \\ 0.0524(\alpha R_{es})^{1/2} & \text{for } R_{es} > 40 \end{cases}$$

$$\alpha = \frac{\dot{\gamma} d}{2|u^f - u^p|} = \frac{R_{es}\varepsilon^2}{2} = \frac{R_{eG}}{2R_{es}}$$

$$\frac{F_L}{F_{L(Saff)}} = 0.3\{1+\tanh[2.5\log_{10}(\varepsilon+0.191)]\}\{0.667+\tan[6(\varepsilon-0.32)]\}$$

$$0.1 \le \varepsilon \le 20$$

Lift Force

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McLaughlin (1991)

$$\frac{F_L}{F_{L(Saff)}} = \begin{cases} 1-0.287\varepsilon^{-2} & \text{for } \varepsilon >> 1 \\ -140\varepsilon^5 \ln(\varepsilon^{-2}) & \text{for } \varepsilon << 1 \end{cases}$$

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Lift Force - Wall Effect

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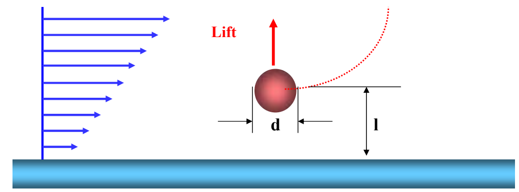
Cherukat and McLaughlin (1994)



$$F_{L(C-L)} = \rho V^2 d^2 I_L / 4$$

$$V = u^p - u^f = u^p - \dot{\gamma} l$$

$$I_L = I_L(\Lambda_G, K)$$

$$\Lambda_G = \frac{\dot{\gamma} d}{2V}$$

$$K = \frac{d}{2l}$$

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Lift Force on a Particle Touching a Plane

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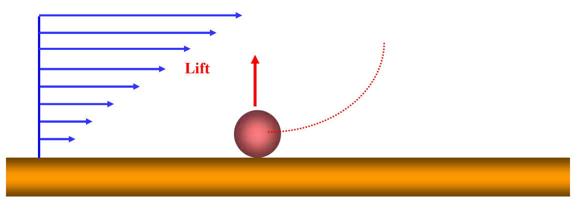

Leighton and Acrivos (1985)

$$F_{L(L-A)} = 0.576\rho d^4 \dot{\gamma}^2$$

Saffman

$$F_{L(Saff)} = 0.807\rho \nu^{1/2} d^3 \dot{\gamma}^{3/2}$$

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Lift Force in Turbulent Boundary Layer

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Velocity Field in the Inertial Sublayer

$$u^+ = \frac{1}{\kappa} \ln y^+ + B$$

$$B \approx 5$$

$$30 < y^+ \le 300$$

Wall Units

$$u^+ = \frac{u}{u^*}$$

$$y^+ = \frac{u^* y}{\nu}$$

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Viscous Sublayer

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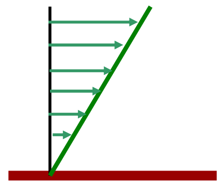
Turbulent stress is negligible

$$\tau_0 = \mu \frac{dU}{dy}$$

$$u^{*2} = \nu \frac{dU}{dy}$$



$$\frac{dU^+}{dy^+} = 1$$

$$u^+ = y^+$$

$$0 < y^+ \le 5$$

Velocity Near a Wall

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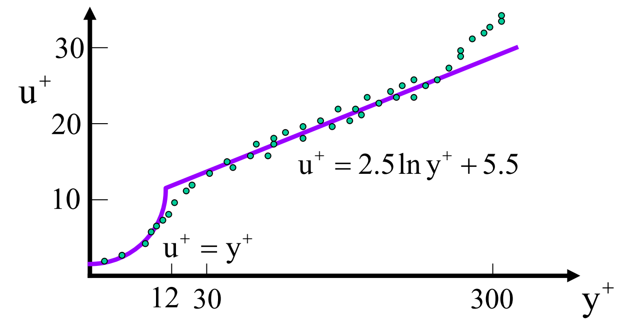

Lift Force in Turbulent Boundary Layer

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$$\gamma = \frac{u^{*2}}{\nu}$$

$$F_L^+ = \frac{F_L}{\rho \nu^2}$$

$$d^+ = \frac{du^*}{\nu}$$

$$F^+_{L(L-A)} = 0.576 d^{+4}$$

$$F^+_{L(Saff)} = 0.807 d^{+3}$$

Hall (1988)

$$F^+_{L(Hall)} = 4.21 d^{+2.31}$$

Mollinger and Nieuwstadt (1996)

$$F^+_{L(MN)} = 15.57 d^{+1.87}$$

Lift Force in Turbulent Boundary Layer

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