ME 529 - Stochastics

Several Random Variables

Goodarz Ahmadi

Department of Mechanical and Aeronautical Engineering
Clarkson University
Potsdam, NY 13699-5725

Several Random Variables

Outline

- Several Random Variables
- Joint Distribution, Density Functions
- Independent Random Variables
- Expected Value
- Covariance
- Joint Characteristic Function

Several Random Variables

Given a probability experiment $\Im$: (S, F, P), a random vector $X(\xi) = (X_1(\xi), X_2(\xi), \ldots, X_n(\xi))$ is defined as a mapping of the probability space unto a point of the *n*-dimensional Euclidean space R^n. That is $X(\xi)$ is defined by a certain rule for every $\xi \in S$.

Joint Distribution and Density Functions

Joint Distribution Function

$$F_X(x_1,\ldots,x_n) = P\{X_1 \le x_1,\ldots,X_n \le x_n\}$$

Joint Density Function

$$f_X(x_1,\ldots,x_n) = \frac{\partial^n F_X(x_1,\ldots,x_n)}{\partial x_1 \ldots \partial x_n}$$

Properties

$$F_X(\infty,\infty,\ldots,\infty) = \int_{-\infty}^{+\infty}\ldots\int_{-\infty}^{+\infty} f_X(x_1,\ldots,x_n)dx_1\ldots dx_n = 1$$

$$P\{X_1(\xi),\ldots,X_n(\xi) \in D\} = \int_D \ldots \int f(x_1,x_2,\ldots,x_n)dx_1\ldots dx_n$$

Independent Random Variables

Clarkson University

The random variables $X_1, X_2, \ldots, X_n$ are said to be independent if the events $\{X_1 \le x_1\}, \ldots, \{X_n \le x_n\}$ are independent for any $x_1, \ldots, x_n$.

$$F_{\mathbf{X}}(x_1, x_2, \ldots, x_n) = F_1(x_1)F_2(x_2)\ldots F_n(x_n)$$

$$f_x(x_1, x_2, \ldots, x_n) = f_1(x_1)f_2(x_2)\ldots f_n(x_n)$$

Several Random Variables

Clarkson University

Expected Value

$$E\{g(X_1,\ldots,X_n)\} = \int_{-\infty}^{+\infty}\ldots\int_{-\infty}^{+\infty} g(x_1,\ldots,x_n) f_{\mathbf{X}}(x_1,\ldots,x_n)dx_1\ldots dx_n$$

Covariance

$$c_{ij} = E\{(X_i - \eta_i)(X_j - \eta_j)\} = E\{X_i X_j\} - \eta_i \eta_j$$

$$\eta_i = E\{X_i\}$$

Joint Characteristic Function

Clarkson University

$$\Phi_{\mathbf{X}}(\omega_1,\ldots,\omega_n) = E\left\{e^{i(\omega_1 X_1 + \ldots + \omega_n X_n)}\right\} = E\left\{e^{i\boldsymbol{\omega}\cdot\mathbf{X}}\right\}$$

Characteristic and Density Function of Fourier Transform Pair

$$\Phi_{\mathbf{X}}(\boldsymbol{\omega}) = \int_{-\infty}^{+\infty}\ldots\int_{-\infty}^{+\infty} e^{i\boldsymbol{\omega}\cdot\mathbf{x}} f_{\mathbf{X}}(\mathbf{x})dx_1\ldots dx_n$$

$$f_{\mathbf{X}}(\mathbf{x}) = \frac{1}{(2\pi)^n}\int_{-\infty}^{+\infty}\ldots\int_{-\infty}^{+\infty} e^{-i\boldsymbol{\omega}\cdot\mathbf{x}}\Phi_{\mathbf{x}}(\boldsymbol{\omega})d\omega_1\ldots d\omega_n$$

Several Random Variables

Clarkson University

If $X_1, X_2, \ldots, X_n$ are independent random variables

$$\Phi(\omega_1,\ldots,\omega_n) = \Phi_1(\omega_1)\ldots\Phi_n(\omega_n)$$

Concluding Remarks

- Several Random Variables
- Joint Distribution, Density Functions
- Independent Random Variables
- Expected Value
- Covariance
- Joint Characteristic Function

Several Random Variables

Clarkson University

Thank you!

Questions?

ME 529 - Stochastics G. Ahmadi