Inviscid Shear Flows

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Inviscid Flows

Outline

- **Inviscid sheared flows**
- **Flow over a cylinder in slightly sheared flows**
- **Perturbation solution**

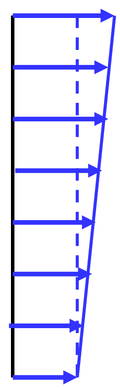
Inviscid Shear Flows

Flow with a slight shear

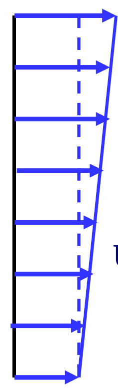
$$u = U_\infty \left(1 + \frac{\varepsilon y}{R}\right)$$

$$\psi = U_\infty \left(y + \frac{\varepsilon y^2}{2R}\right)$$

$$\omega = -\nabla^2 \psi = -\frac{\partial u}{\partial y} = -\frac{\varepsilon U_\infty}{R}$$



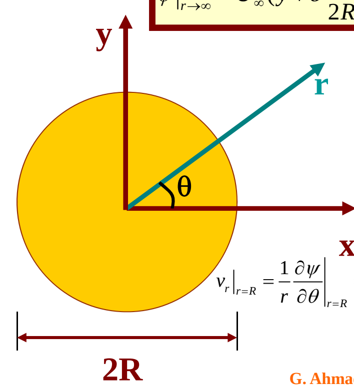
Sheared Flows around a Cylinder



$$\nabla^2 \psi = -\omega$$

U=U$_\infty$(1+εy/R)

$$\psi\big|_{r\to\infty} = U_\infty \left(y + \varepsilon \frac{y^2}{2R}\right)$$



$$v_r\big|_{r=R} = \frac{1}{r}\frac{\partial \psi}{\partial \theta}\Big|_{r=R} = 0$$

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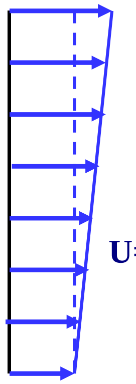

$U=U_\infty(1+\varepsilon y/R)$

$$\nabla^2\psi = -\omega = \frac{\varepsilon U_\infty}{R}$$

$$\psi\big|_{r\to\infty} = U_\infty\left(y+\frac{\varepsilon y^2}{2R}\right)$$
$$= U_\infty\left[r\sin\theta + \frac{\varepsilon r^2}{4R}(1-\cos 2\theta)\right]$$

$$v_r\big|_{r=R} = \frac{1}{r}\frac{\partial\psi}{\partial\theta}\bigg|_{r=R} = 0$$

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Perturbation solution

$$\psi = \psi_o + \varepsilon\psi_1 + \varepsilon^2\psi_2 + \ldots$$

$$\nabla^2\psi_o + \varepsilon\nabla^2\psi_1 = \frac{\varepsilon U_\infty}{R}$$

$$\varepsilon^0 \rightarrow \nabla^2\psi_o = 0$$

BC

$$\psi_o\big|_{r\to\infty} = U_\infty r\sin\theta$$

$$\frac{1}{r}\frac{\partial\psi_o}{\partial\theta}\bigg|_{r=R} = 0$$

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Perturbation solution

$$\varepsilon^1 \rightarrow \nabla^2\psi_1 = \frac{U_\infty}{R}$$

BC

$$\psi_1\big|_{r\to\infty} = \frac{U_\infty r^2}{4R}(1-\cos 2\theta)$$

$$\frac{\partial\psi_1}{\partial\theta}\bigg|_{r=R} = 0$$

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Perturbation Solution

$$\nabla^2\psi_o = 0$$

BC

$$\psi_o\big|_{r\to\infty} = U_\infty r\sin\theta$$

$$\frac{1}{r}\frac{\partial\psi_o}{\partial\theta}\bigg|_{r=R} = 0$$

$$\psi_o = U_\infty\left(r-\frac{a^2}{r}\right)\sin\theta$$

$$\psi_o\big|_{r=R} = 0$$

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Perturbation solution

$$\nabla^2\psi_1 = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \psi_1}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 \psi_1}{\partial \theta^2} = \frac{U_\infty}{R}$$

Let

$$\psi_1 = \frac{U_\infty r^2}{4R}(1-\cos 2\theta) + \varphi_1$$

$$\frac{U_\infty}{R}(1-\cos 2\theta) + \frac{U_\infty}{R}\cos 2\theta + \nabla^2\varphi_1 = \frac{U_\infty}{R}$$

$$\nabla^2\varphi_1 = 0$$

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$$\nabla^2\varphi_1 = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \varphi_1}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 \varphi_1}{\partial \theta^2} = 0$$

BC

$$\varphi_1\big|_{r\to\infty} = Const$$

$$\varphi_1\big|_{r=R} = -\frac{U_\infty R}{4}(1-\cos 2\theta)$$

Let

$$\varphi_1 = \frac{U_\infty R}{4}[-1 + g(r)\cos 2\theta]$$

BC

$$\nabla^2\varphi_1 = \left[\frac{1}{r}\frac{d}{dr}\left(r\frac{dg}{dr}\right) - \frac{4g}{r^2}\right]\cos 2\theta = 0$$

$$\to \quad \frac{d^2 g}{dr^2} + \frac{1}{r}\frac{dg}{dr} - \frac{4g}{r^2} = 0$$

$$g\big|_{r\to\infty} = U_\infty r$$

$$g\big|_{r=R} = 1$$

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Let $g = Br^m$

$$m(m-1) + m - 4 = 0 \qquad m^2 - 4 = 0 \to \quad m = \pm 2$$

$$g = Ar^2 + \frac{B}{r^2} \qquad A = 0 \qquad B = R^2$$

$$g = \frac{R^2}{r^2}$$

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$$\varphi_1 = \frac{U_\infty R}{4}\left[-1 + \frac{R^2\cos 2\theta}{r^2}\right]$$

$$\psi_1 = \frac{U_\infty R}{4}\left[\frac{r^2}{R^2}(1-\cos 2\theta) - 1 + \frac{R^2\cos 2\theta}{r^2}\right]$$

Perturbation Solution

$$\psi = U_\infty\left(r - \frac{a^2}{r}\right)\sin\theta + \frac{\varepsilon U_\infty R}{4}\left[\frac{r^2}{R^2}(1-\cos 2\theta) - 1 + \frac{R^2\cos 2\theta}{r^2}\right]$$

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Concluding Remarks

- Inviscid sheared flows
- Flow over a cylinder in slightly sheared flows
- Perturbation solution

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Thank you!

Questions?

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