ME 326 - Intermediate Fluid Mechanics

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Shock Waves

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Compressible Flows

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Outline

- Compressible Flow Regimes
  - Thermodynamics
  - Speed of Sound & Mach Number
- Isentropic Flows with Area Change
  - Variations with Mach number
- Shock Waves
  - Operation of Nozzle and Diffusers
- Flows with Friction
- Flows with Heat Transfer

Normal Shock Waves

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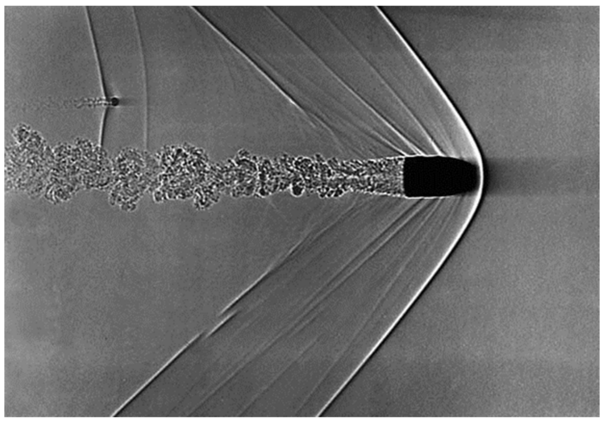

Normal Shock Waves

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Shock → Discontinuity (Sharp Variation) in Flow Properties

→ Shocks Occur in Supersonic Flows

Fanno Line

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Flows with Friction (No Heat Transfer)

Energy Equation ⇒ $$(h_1 + \frac{V_1^2}{2}) = (h_2 + \frac{V_2^2}{2}) = h_o$$

Mass $\rho_1 V_1 = \rho_2 V_2$

Momentum $$P_1 - P_2 + R / A = \rho V (V_2 - V_1)$$

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Fanno Line

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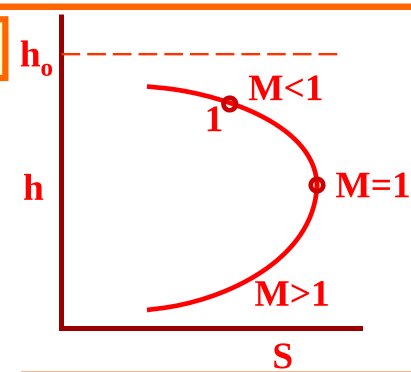
Equation of State

$h = h(S, \rho)$

$\rho = \rho(S, P)$

Select a v_2
Mass ⇒ ρ_2
Energy ⇒ h_2
State ⇒ S_2



Point 2 could be any point on Fanno Line

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Rayleigh Line

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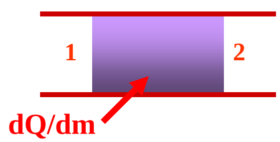
Flows with Heat Transfer (No Friction)

Energy Equation ⇒ $$h_1 + \frac{V_1^2}{2} + \frac{dQ}{dm} = h_2 + \frac{V_2^2}{2}$$

Mass $\rho_1 V_1 = \rho_2 V_2$



Momentum $$P_1 - P_2 = \rho V (V_2 - V_1)$$

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Rayleigh Line

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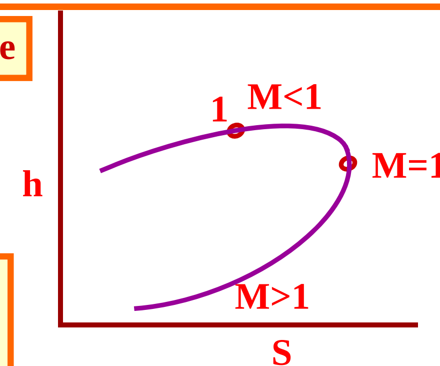
Equation of State

$h = h(S, \rho)$

$\rho = \rho(S, P)$

Select a v_2
Mass ⇒ ρ_2
momentum ⇒ P_2
State ⇒ S_2, h_2



Point 2 could be any point on Rayleigh Line

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Normal Shock Waves

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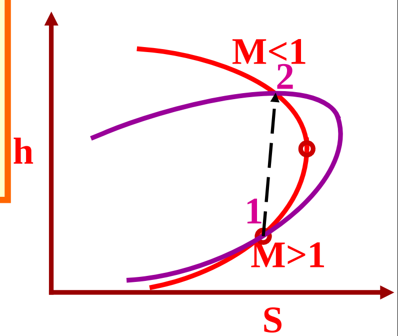
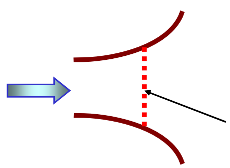
Flow state can change from 1 to 2 with "No Friction" and "No Heat Transfer"
$\Rightarrow$ Shock, $\Delta S > 0$





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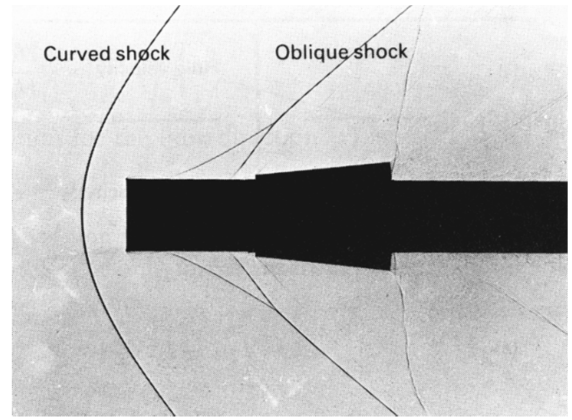

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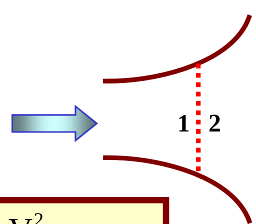
Energy Equation



$$C_pT_{o1} = C_pT_1 + \frac{V_1^2}{2} = C_pT_2 + \frac{V_2^2}{2} = C_pT_{o2}$$

$$T_{o1} = T_{o2}$$

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Temperature Ratio

$$\frac{T_2}{T_1} = \frac{1 + \frac{k-1}{2} M_1^2}{1 + \frac{k-1}{2} M_2^2}$$

Mass

$$\rho_1 V_1 = \rho_2 V_2$$

$$\frac{P_1}{RT_1} M_1 a_1 = \frac{P_2}{RT_2} M_2 a_2$$

Pressure Ratio
Fanno Line

$$\frac{P_2}{P_1} = \frac{M_1}{M_2}\left(\frac{1 + \frac{k-1}{2} M_1^2}{1 + \frac{k-1}{2} M_2^2}\right)^{1/2}$$

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Normal Shock Waves

Momentum

$$P_1 - P_2 = \rho_2 V_2^2 - \rho_1 V_1^2$$

Pressure Ratio
Rayleigh Line

$$\frac{P_2}{P_1} = \frac{1 + kM_1^2}{1 + kM_2^2}$$

Mach Number

$$M_2^2 = \frac{(k-1)M_1^2 + 2}{2kM_1^2 + (1-k)}$$

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Normal Shock Waves

Pressure Ratio

$$\frac{P_2}{P_1} = \frac{2k}{k+1} M_1^2 - \frac{k-1}{k+1}$$

Temperature Ratio

$$\frac{T_2}{T_1} = \frac{\left(1 + \frac{k-1}{2} M_1^2\right)\left(\frac{2k}{k-1} M_1^2 - 1\right)}{\frac{(k+1)^2}{2(k-1)} M_1^2}$$

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Normal Shock Waves

K=1.4

$$M_2^2 = \frac{M_1^2 + 5}{7M_1^2 - 1}$$

$$\frac{P_2}{P_1} = 1.1667M_1^2 - 0.1667$$

$$\frac{T_2}{T_1} = \frac{(1 + 0.2M_1^2)(7M_1^2 - 1)}{7.2M_1^2}$$

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K=1.4

$$T_{o2} = T_{o1}$$

$$\frac{P_{o2}}{P_{o1}} = \left(\frac{1.2M_1^2}{1 + 0.2M_1^2}\right)^{3.5} \left(\frac{1}{1.1666M_1^2 - 0.1667}\right)^{2.5}$$

$$\frac{A_2^*}{A_1^*} = \frac{M_2}{M_1} \left(\frac{1 + 0.2M_1^2}{1 + 0.2M_2^2}\right)^3$$

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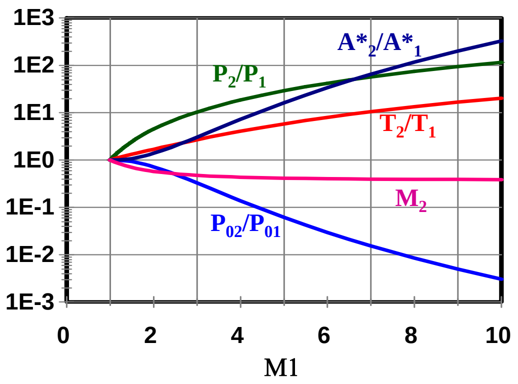

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Operation of Nozzles

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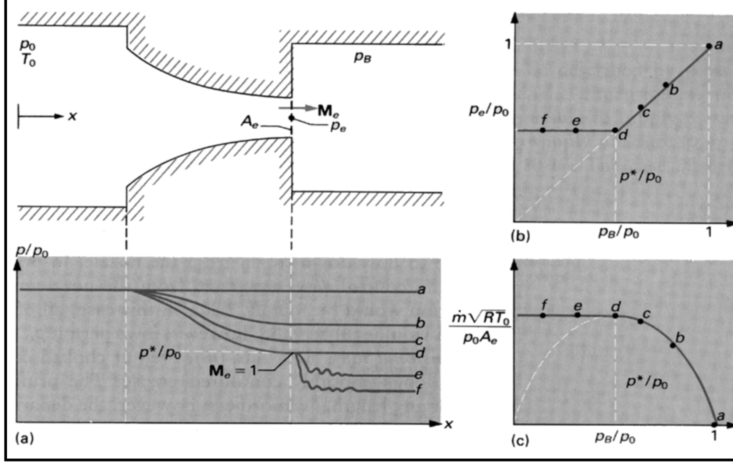

Operation of Nozzles

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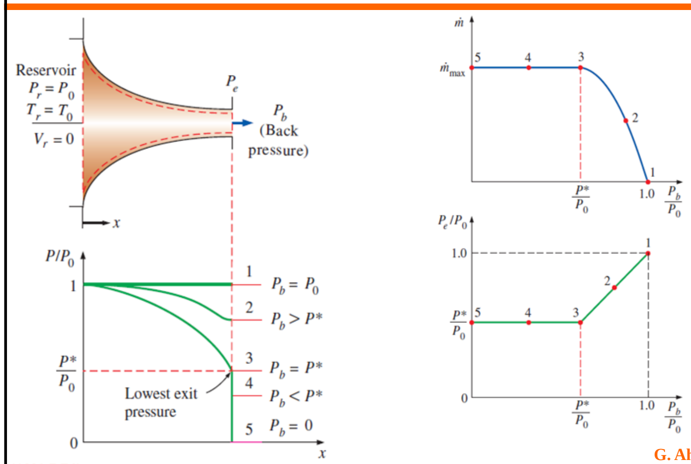

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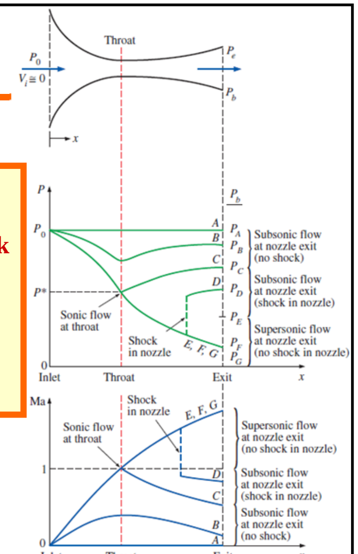
Operation of Nozzles

$P_C<P_b<P_o$	Subsonic Flow
$P_F<P_b<P_C$	Supersonic Flow+ Shock
$P_b=P_F$	Supersonic Flow
$P_b<P_F$	Supersonic Flow+ Shocks out side



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Operation of Nozzles

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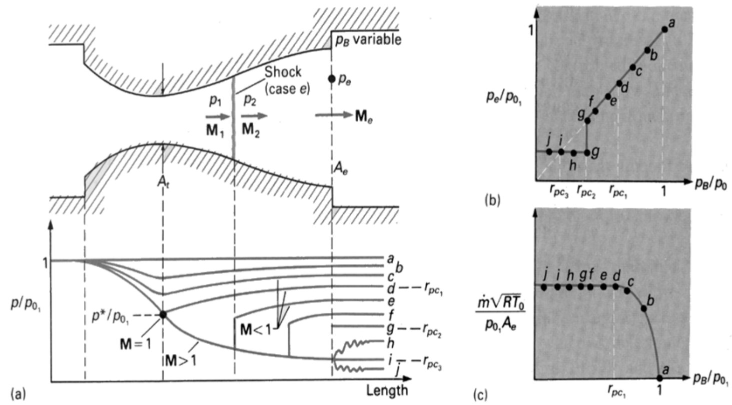

Isoentropic Flows

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Table 1

M	P/P_o	ρ/ρ_o	T/T_o	A/A^*
0	1	1	1	-
0.4	0.896	0.934	0.969	1.59
1	0.528	0.634	0.833	1
2	0.128	0.23	0.556	1.688
4	0.0066	0.0277	0.238	10.71

Normal Shock Waves

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Table 2 To2/To1=1

M_1	M_2	P_2/P_1	ρ_2/ρ_1	T_2/T_1	P_{o2}/P_{o1}	A_2^*/A_1^*
1	1	1	1	1	1	1
1.5	0.701	2.458	1.862	1.320	0/930	1.076
2	0.577	4.5	2.667	1.689	0.721	1.387
3	0.475	10.333	3.857	2.679	0.328	3.04688
4	0.435	18.5	4.571	4.047	0.139	7.207

Compressible Flows

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Concluding Remarks

- Shock Waves
- Fanno & Rayeigh Lines
- Normal Shock Relationships
- Operation of Nozzle and Diffusers