ME 326 - Intermediate Fluid Mechanics

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Flows With Heat Transfer

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Compressible Flows

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Outline

- Compressible Flow Regimes
  - Thermodynamics
  - Speed of Sound & Mach Number
- Isentropic Flows with Area Change
  - Variations with Mach number
- Shock Waves
  - Nozzle and Diffusers
- **Flows with Heat Transfer**
- **Flows with Friction**

Rayleigh Line

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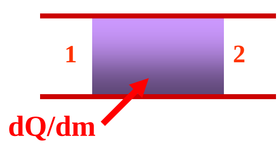
Flows with Heat Transfer (No Friction)

Energy Equation ⇒

$$h_1 + \frac{V_1^2}{2} + \frac{dQ}{dm} = h_2 + \frac{V_2^2}{2}$$

Mass $\rho_1 V_1 = \rho_2 V_2$



Momentum $P_1 - P_2 = \rho V (V_2 - V_1)$

Rayleigh Line

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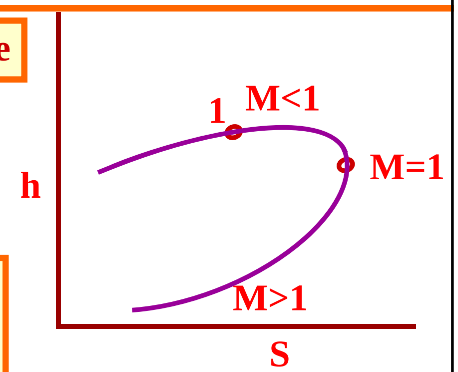
Equation of State

$h = h(S, \rho)$

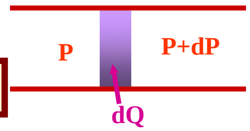
$\rho = \rho(S, P)$

Select a v_2
Mass ⇒ ρ_2
momentum ⇒ P_2
State ⇒ S_2, h_2



Point 2 could be any point on Rayleigh Line

Energy Equation



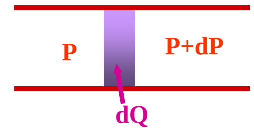
1 $$dQ = C_p dT + VdV$$

Continuity Equation

$$\rho V = \text{Const.}$$

2 $$\frac{d\rho}{\rho} + \frac{dV}{V} = 0$$

Momentum Equation



$$PA - (P + dP)A = \rho VA(V + dV - V)$$

3 $$dP + \rho VdV = 0$$

Equation of State

$$P = \rho RT$$

4 $$\frac{dP}{P} = \frac{d\rho}{\rho} + \frac{dT}{T}$$

Mach Number

$$M^2 = \frac{V^2}{kRT}$$

5 $$\frac{2dM}{M} = \frac{2dV}{V} - \frac{dT}{T}$$

5 Equations for 5 Unknowns

$$\frac{dM}{M}, \frac{dV}{V}, \frac{dT}{T}, \frac{dP}{P}, \frac{d\rho}{\rho}$$

$$\frac{dP}{P} = \frac{-kM^2}{1 - M^2} \frac{dQ}{C_p T}$$

$$\frac{dM^2}{M^2} = \frac{1 + kM^2}{1 - M^2} \frac{dQ}{C_p T}$$

Flows with Heat Transfer

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The effects of heating and cooling on the properties of Rayleigh flow

	Heating	
Property	Subsonic	Supersonic
Velocity, V	Increase	Decrease
Mach number, Ma	Increase	Decrease
Stagnation temperature, T_0	Increase	Increase
Temperature, T	Increase for Ma $< 1/k^{1/2}$ Decrease for Ma $> 1/k^{1/2}$	Increase
Density, ρ	Decrease	Increase
Stagnation pressure, P_0	Decrease	Decrease
Pressure, P	Decrease	Increase
Entropy, s	Increase	Increase

Flows with Heat Transfer

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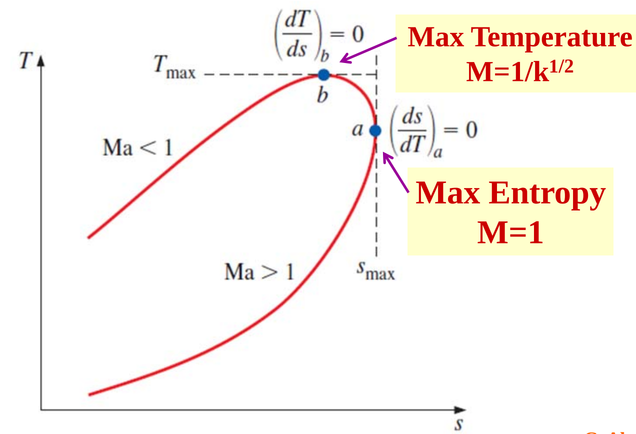

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Property Ratios

$$\frac{P}{P^*} = \frac{k+1}{1+kM^2}$$

$$\frac{T}{T^*} = \left(\frac{(k+1)M}{1+kM^2}\right)^2$$

$$\frac{V}{V^*} = \frac{(k+1)M^2}{1+kM^2} = \frac{\rho^*}{\rho}$$

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Property Ratios

$$\frac{P_o}{P_o^*} = \frac{k+1}{1+kM^2}\left(\frac{2+(k-1)M^2}{k+1}\right)^{k/(k-1)}$$

$$\frac{T_o}{T_o^*} = \frac{(k+1)M^2[2+(k-1)M^2]}{(1+kM^2)^2}$$

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Table 4

M	T/T*	P/P*	P_o/P_o*	V/V*	T_o/T_o*
0	0	2.4	1.268	0	0
0.02	0.0023	2.399	1.267	0.001	0.0019
0.5	0.790	1.777	1.114	0.0.444	0.691
1	1	1	1	1	1
3	0.280	0.176	3.42	1.588	0.654

Compressible Flows

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Concluding Remarks

- Compressible Flows with Heat Transfer
- Rayleigh Line
- Variation of Property Ratios with Mach Number
- Table 4

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Thank you!

Questions?