ME 639 EXAM 1 April 2004

1) (20 Points) Consider the turbulent flow of an incompressible fluid with Pr=1. Estimate the order of magnitude of the following quantities in terms of u, λ, θ and Λ:

$$\text{a) } \overline{u'_k \frac{\partial u'_k}{\partial x_i} u'_i} \qquad \text{b) } \overline{\frac{\partial^2 \omega'_i}{\partial x_j \partial x_j} \frac{\partial^2 \omega'_i}{\partial x_k \partial x_k}} \qquad \text{c) } \overline{\omega'_i \frac{\partial u'_j}{\partial x_i} u'_j}$$

a) $\frac{\partial}{\partial x_i}\left(\overline{u'_i \frac{u'_k u'_k}{2}}\right) \sim \frac{u^3}{\Lambda}$

$\frac{v}{\eta} = \frac{u}{\lambda}$ $\qquad$ $\eta^2 \Lambda = \lambda^3$

$\nu \sim \frac{u\lambda^2}{\Lambda}$

b) $\overline{\frac{\partial^2 \omega'_i}{\partial x_j \partial x_j} \frac{\partial^2 \omega'_i}{\partial x_j \partial x_j}} \sim \frac{v^2}{\eta^2 \eta^4} = \frac{u^2}{\lambda^2 \eta^4}$

$= \frac{u^2 \Lambda^2}{\lambda^2 \lambda^6} = \frac{u^2 \Lambda^2}{\lambda^8}$

c) $\overline{\omega'_i \frac{\partial}{\partial x_i} \frac{u'_j u'_j}{2}} = \frac{\partial}{\partial x_i}\left(\overline{\omega'_i \frac{u'_j u'_j}{2}}\right) = \frac{1}{\Lambda} \frac{u^3}{\Lambda} = \frac{u^3}{\Lambda^2}$

$$\text{d) } \overline{\omega_i'\omega_j' \frac{\partial u_i'}{\partial x_k}\frac{\partial u_j'}{\partial x_k}} \qquad \text{e) } \overline{\frac{\partial u_i'}{\partial x_k}\frac{\partial u_j'}{\partial x_k}} D_{ij} \qquad \text{f) } \overline{\frac{\partial \omega_k'}{\partial x_j}\frac{\partial^2 \omega_i'}{\partial x_i \partial x_k}}$$

$$\text{d) } \overline{\omega_i'\omega_j' \frac{\partial u_i'}{\partial x_k}\frac{\partial u_j'}{\partial x_k}} \sim \frac{u^4}{\lambda^4} - \left(\frac{v^4}{\eta^4}\right)$$

$$\text{e) } \overline{\frac{\partial u_i'}{\partial x_k}\frac{\partial u_j'}{\partial x_k}} D_{ij} \sim \frac{u^2}{\lambda^2}\left(\alpha\delta_{ij} + b_{ij}\frac{\lambda}{\Lambda}\right) D_{ij} \sim \frac{u^2}{\lambda^2}\frac{\lambda}{\Lambda}\frac{u}{\Lambda} - \frac{u^3}{\lambda\Lambda^2}$$

$$\text{f) } \overline{\frac{\partial \omega_k'}{\partial x_j}\frac{\partial^2 \omega_i'}{\partial x_i \partial x_k}} = 0 \qquad \frac{\partial \omega_i'}{\partial x_i} = 0$$

2) (30 Points) The Burger equations for momentum and hear transfer are given as

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \nu\frac{\partial^2 u}{\partial x^2} \qquad \frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} = \nu\frac{\partial^2 T}{\partial x^2}$$

Derive the transport equation for the velocity-temperature correlation $\overline{u'T'}$ **for the burger model. Identify the terms corresponding to production, dissipation, diffusion and convection. Find the order of magnitude of different terms.**

$u = \bar{u} + u'$, $T = \bar{T} + T'$

$$\frac{\partial \bar{u}}{\partial t} + \bar{u}\frac{\partial \bar{u}}{\partial x} = -\frac{\partial}{\partial x}\frac{\overline{u'^2}}{2} + \nu\frac{\partial^2 \bar{u}}{\partial x^2}$$

$$\frac{\partial \bar{T}}{\partial t} + \bar{u}\frac{\partial \bar{T}}{\partial x} = -\overline{u'\frac{\partial T'}{\partial x}} + \nu\frac{\partial^2 \bar{T}}{\partial x^2}$$

subtracting

$$T'\left(\frac{\partial u'}{\partial t} + \bar{u}\frac{\partial u'}{\partial x} + u'\frac{\partial \bar{u}}{\partial x} + u'\frac{\partial u'}{\partial x} = \nu\frac{\partial^2 u'}{\partial x^2} + \frac{\partial}{\partial x}\frac{\overline{u'^2}}{2}\right.$$

$$u'\left(\frac{\partial T'}{\partial t} + \bar{u}\frac{\partial T'}{\partial x} + u'\frac{\partial \bar{T}}{\partial x} + u'\frac{\partial T'}{\partial x} = \nu\frac{\partial^2 T'}{\partial x^2} + \overline{u'\frac{\partial T'}{\partial x}}\right.$$

$$\frac{\partial \overline{u'T'}}{\partial t} + \bar{U}\frac{\partial \overline{u'T'}}{\partial x} = -\overline{u'T'}\frac{\partial \bar{U}}{\partial x} - \overline{u'^2}\frac{\partial \bar{T}}{\partial x} \qquad \frac{u^2\theta}{\Lambda}$$

convection — $\frac{u^2\theta}{\Lambda}$ — Grad. production

$$-\overline{T'u'\frac{\partial u'}{\partial x}} - \overline{u'^2\frac{\partial T'}{\partial x}} + \nu\frac{\partial^2 \overline{u'T'}}{\partial x^2} \quad \text{viscous Diff.}$$

$$\frac{u^2\theta}{\Lambda} \qquad \nu\frac{u\theta}{\Lambda^2} \rightarrow \frac{u^2\lambda^2\theta}{\Lambda^3}$$

$$-2\nu\,\overline{\frac{\partial u'}{\partial x}\frac{\partial T'}{\partial x}}$$

Dissipation

$$\nu\frac{u}{\lambda}\frac{\theta}{\lambda_\theta} = \frac{\nu u\theta}{\lambda^2}$$

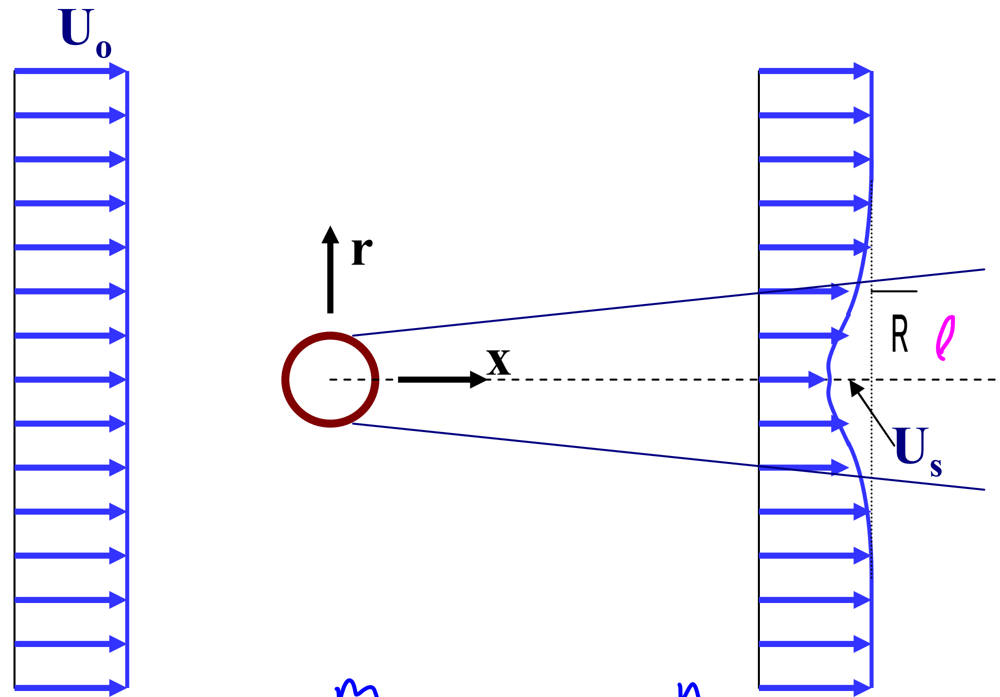
$$-\overline{T'\frac{\partial}{\partial x}\frac{u'^2}{2}} - \overline{u'^2\frac{\partial T'}{\partial x}} = -\frac{\partial}{\partial x}\overline{T'\frac{u'^2}{2}} - \overline{\frac{u'^2}{2}\frac{\partial T'}{\partial x}} \qquad -\frac{u\lambda^2}{\Lambda}\frac{u\theta}{\lambda^2} - \frac{u^2\theta}{\Lambda}$$

Turb. Diff — sonic

$$\nu - \frac{u\lambda^2}{\Lambda}$$

3) (25 Points) For a laminar axisymmetric wake flow behind a sphere, obtain the variations of Us and ℓ with x. The equation governing of motion is given as

$$U_o \frac{\partial U}{\partial x} = \frac{\nu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial U}{\partial r}\right)$$



$U_0 - U = f(\eta)$ $\quad \eta = r/\ell \quad$ $U_s \sim A x^m$, $\ell \sim B x^n$

power of x $\quad \cancel{m} - 1 = \cancel{m} - 2n \Rightarrow n = 1/2$

$$M = 2\pi U_0 \int_0^\infty (U - U_0)\, r\, dr = \text{const}$$

$m + 2n = 0 \qquad m = -2n = -1$

$\ell = B x^{1/2} \qquad U_s = A x^{-1}$

4) (25 Points) Determine the contribution of eddies of size r to the correlations

a) $\omega_i'\omega_j'\dfrac{\partial u_i'}{\partial x_k}u_j'$ b) $\dfrac{\partial \omega_k'}{\partial x_j}\dfrac{\partial \omega_k'}{\partial x_j}$. For r being Λ , λ , and η evaluate the these contribution and compare.

$$r,\quad v(r) \sim (\varepsilon r)^{1/3}$$

$$\varepsilon = u^3/\Lambda$$

a) $$A = \overline{\omega_i'\omega_j'\frac{\partial u_i'}{\partial x_k}u_j'} \sim \frac{v(r)^4}{r^3} = \frac{(\varepsilon r)^{4/3}}{r^3}$$

$$\sim \frac{(u^3 r/\Lambda)^{4/3}}{r^3} = \frac{u^4}{\Lambda^{4/3} r^{3-4/3}} = \frac{u^4}{\Lambda^{4/3} r^{5/3}}$$

$$\eta^2\Lambda = \lambda^3$$

$$\eta^2 = \lambda^3/\Lambda$$

$r = \Lambda$ $$A \sim \frac{u^4}{\Lambda^3}$$

$r = \lambda$ $$A \sim \frac{u^4}{\Lambda^{4/3}\lambda^{5/3}}$$

$r = \eta$ $$A \sim \frac{u^4}{\Lambda^{4/3}\eta^{5/3}} \sim \frac{u^4}{\Lambda^{4/3}(\lambda^3/\Lambda)^{5/6}} \sim \frac{u^4}{\Lambda^{1/2}\lambda^{5/2}}$$

b) $$B - \overline{\frac{\partial \omega_k'}{\partial x_j}\frac{\partial \omega_k'}{\partial x_j}} - \frac{\nu(r)^2}{r^4} - \frac{(\varepsilon r)^{2/3}}{r^4} - \frac{(u^3 r/\Lambda)^{2/3}}{r^4}$$

$$B - \frac{u^2}{\Lambda^{2/3} r^{10/3}}$$

$$\eta = \frac{\lambda^{3/2}}{\Lambda^{1/2}}$$

$r = \Lambda$ $$B - \frac{u^2}{\Lambda^4}$$

$r = \lambda$ $$B - \frac{u^2}{\Lambda^{2/3}\lambda^{10/3}}$$

$r = \eta$ $$B - \frac{u^2}{\Lambda^{2/3}\eta^{10/3}} - \frac{u^2}{\Lambda^{2/3}\left(\frac{\lambda^{3/2}}{\Lambda^{1/2}}\right)^{10/3}} = \frac{u^2}{\Lambda^{2/3-5/3}\lambda^5}$$

$$- \frac{u^2\Lambda}{\lambda^5} - \frac{u^2}{\lambda^2\eta^2}$$