ME 639 - Turbulence

Anisotropic Rate-Dependent Turbulence Model

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Rate-Dependent Turbulence Model

Outline

- Averaged Conservation laws
- Entropy Constraints
- Thermodynamics of Turbulence
- Constitutive Equations
- Rate Dependent Model
- Model Predictions
- Comparison with Experimental Data

Averaged Balance Laws

Mass

$$v_{i,i} = 0$$

Momentum

$$\rho\dot{v}_i = t_{ji,j} + t^T_{ji,j} + \rho f_i$$

Thermal Energy →

$$\rho\dot{e} = q_{i,i} + q^T_{i,i} + t_{ij} v_{j,i} + \rho\varepsilon + r$$

Fluctuation Energy →

$$\rho\dot{k} = t^T_{ij} v_{j,i} + K_{i,i} - \rho\varepsilon$$

Entropy

$$\rho\dot{\eta} - (q_i\vartheta)_{,i} - R^T_{i,i} - r\vartheta + \rho\dot{\eta}^T - S^T_{i,i} \ge 0$$

Thermodynamics of Turbulence

Helmholtz Free Energy →

$$\psi = e - \frac{\eta}{\vartheta} \qquad \psi^T = k - \frac{\eta^T}{\vartheta^T}$$

Heat Flux-Coldness →

$$R^T_i = q^T_i \vartheta$$

Energy Flux-Coldness →

$$S^T_i = K_i \vartheta^T - E_i$$

Heat Flux →

$$Q_i = q_i + q^T_i$$

Clausius-Duhem Inequality

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$$\vartheta\left[-\rho\left(\dot\psi-\frac{\eta\dot\vartheta}{\vartheta^2}\right)-\frac{1}{\vartheta}Q_i\vartheta_{,i}+t_{ij}v_{j,i}+\rho\varepsilon\right]$$
$$+\vartheta^T\{-\rho\left[\dot\psi^T-\frac{\eta^T\dot\vartheta^T}{(\vartheta^T)^2}\right]-\frac{1}{\vartheta^T}K_i\vartheta^T_{,i}$$
$$+\frac{1}{\vartheta^T E_{i,i}}+t^T_{ij}v_{j,i}-\rho\varepsilon\}\ge 0$$

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Constitutive Equations Stress Tensors

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$$t^T_{ij}=-\frac{2}{3}\rho k\delta_{ij}+\rho\frac{\partial\psi^T}{\partial\Delta}\frac{\hat{D}d_{ij}}{Dt}$$
$$+\mu^T\{(2+\gamma\tau^2 d_{kl}d_{kl})d_{ij}$$
$$+\beta\tau\left[\frac{1}{3}d_{kl}d_{kl}\delta_{ij}-d_{ik}d_{kj}\right]\}$$

$$t_{ij}=-p\delta_{ij}+2\mu d_{ij}$$

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Constitutive Equations

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Jaumann Derivative →
$$\frac{\hat{D}d_{ij}}{Dt}=\dot d_{ij}+d_{ik}\omega_{kj}+d_{jk}\omega_{ki}$$

$$d_{ij}=\frac{1}{2}(v_{i,j}+v_{j,i})\qquad \omega_{ij}=\frac{1}{2}(v_{i,j}-v_{j,i})\qquad \Delta=\frac{1}{2}d_{ij}d_{ij}$$

Heat Flux
$$Q_i=\left(\kappa+C\frac{\mu^T}{\sigma^\theta}\right)\theta_{,i}$$

Energy Flux
$$K_i=\left(\mu+\frac{\mu^T}{\sigma^k}\right)\left[k_{,i}-\frac{k}{\tau}\tau_{,i}\right]$$

Heat Capacity
$$C=-\theta\frac{\partial^2\psi}{\partial\theta^2}$$

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Thermodynamics Constraints

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$$\mu^T\ge 0\qquad \sigma^\theta\ge 0\qquad \gamma\ge 0\qquad |\beta|^2\le 48\gamma$$

Eddy Viscosity →
$$\mu^T=C^\mu\rho k\tau$$

Turbulence Free Energy →
$$\psi^T=k\left[\ln\left(\frac{\tau^{\alpha_0}}{k}\right)+\alpha C^\mu\tau^2\Delta+C_0\right]$$

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Constitutive Equations Turbulence Stress Tensor

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$$t^T_{ij} = -\frac{2}{3}\rho k \delta_{ij} + \mu^T\{2d_{ij} + \alpha\tau\frac{\hat{D}}{Dt}d_{ij} + \gamma\tau^2 d_{lk}d_{kl}d_{ij} + \beta\tau\left[\frac{1}{3}d_{lk}d_{kl}\delta_{ij} - d_{ik}d_{kj}\right]\}$$

Governing Equations

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$$v_{i,i} = 0$$

$$\rho\dot{v}_i = -\left[p + \frac{2}{3}\rho k\right]_{,i} + \{2(\mu+\mu^T)d_{ij} + \mu^T[\alpha\tau\frac{\hat{D}d_{ij}}{Dt} + \beta\tau(\frac{1}{3}d_{lk}d_{kl}\delta_{ij} - d_{ik}d_{kj}) + \gamma\tau^2 d_{lk}d_{kl}d_{ij}]\}_{,j} + \rho f_i$$

Governing Equations

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$$\rho C\dot{\theta} = \left[(\kappa + C\frac{\mu^T}{\sigma^\theta})\theta_{,i}\right]_{,i} + 2\mu d_{ij}d_{ij} + \rho\varepsilon + r$$

$$\rho\dot{k} = \left[(\mu + \frac{\mu^T}{\sigma^k})(k_{,i} - \frac{k}{\tau}\tau_{,i})\right]_{,i} + P + \alpha\tau\mu^T\frac{\hat{D}d_{ij}}{Dt}d_{ij} - \rho\varepsilon$$

$$P = \mu^T[2d_{ij}d_{ij} - \beta\tau d_{ik}d_{kj}d_{ij} + \gamma\tau^2(d_{ij}d_{ji})^2]$$

Scale Transport Equation

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$$\rho\dot{\tau} = \left[(\mu + \frac{\mu^T}{\sigma^T})\tau_{,i}\right]_{,i} + C^{\tau_1}\frac{\tau}{k}P + C^{\tau_3}\left(\mu + \frac{\mu^T}{\sigma^k}\right)\left(\frac{\tau}{k^2}\right)[k_{,i} - \frac{k}{\tau}\tau_{,i}][k_{,i} - \frac{k}{\tau}\tau_{,i}] + \left(\mu + \frac{\mu^T}{\sigma^T}\right)[\frac{2\alpha C^\mu}{\alpha_0 + 2\alpha C^\mu\tau^2\Delta}](\tau^2\Delta)_{,i}\tau_{,i} - \rho C^{\tau_2}C^D$$

$$\varepsilon = C^D\frac{k}{\tau}$$

$$\frac{1}{\alpha_{0m}} \geq C^{\tau_1} \geq 0 \qquad C^{\tau_2} \geq \frac{1}{\alpha_0} \qquad \frac{1}{\alpha_{0m}} \geq C^{\tau_3} \geq 0 \qquad \alpha_0 \geq 0$$

Scale Transport Equation

Clarkson University

$$\rho\dot{\varepsilon} = \left[(\mu + \frac{\mu^T}{\sigma^\varepsilon})\varepsilon_{,i}\right]_{,i} + C^{\varepsilon_1}\frac{\varepsilon}{k}P + C^{\varepsilon_3}\left(\mu + \frac{\mu^T}{\sigma^k}\right)\left(\frac{\varepsilon}{k^2}\right)[k_{,i} - \frac{k}{\varepsilon}\varepsilon_{,i}][k_{,i} - \frac{k}{\varepsilon}\varepsilon_{,i}]$$

$$+\left(\mu + \frac{\mu^T}{\sigma^\varepsilon}\right)\left[\frac{2\alpha C^\mu}{\alpha_0 + 2\alpha C^\mu \Delta \frac{k^2}{\varepsilon^2}}\right](\Delta\frac{k^2}{\varepsilon^2})_{,i}\varepsilon_{,i} - \rho C^{\varepsilon 2} C^{\varepsilon_2}\frac{\varepsilon^2}{k}$$

$$\mu^T = \rho C^\mu \frac{k^2}{\varepsilon}$$

$$\tau = \frac{k}{\varepsilon}$$

$C^\mu = 0.09$ $\quad\alpha = 0.93$

$C^{\varepsilon 2} = 1.92$ $\quad\beta = 0.54$

$\sigma^k = 1$ $\quad\sigma^\varepsilon = 1.3$ $\quad C^{\varepsilon 1} = 1.45$ $\quad\gamma = 0.005$

Duct Flows

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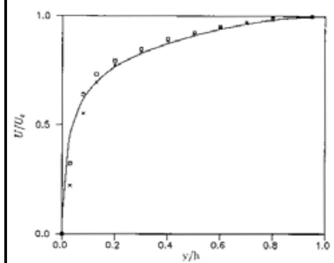
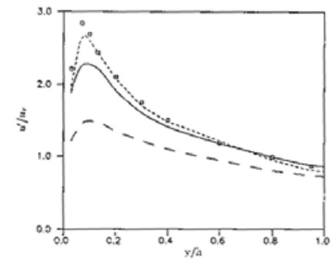


Mean velocity



Axial turbulence intensity

Comparison are with the experimental data of Kreplin and Eckelmann and DNS of Kim et al.

Duct Flows

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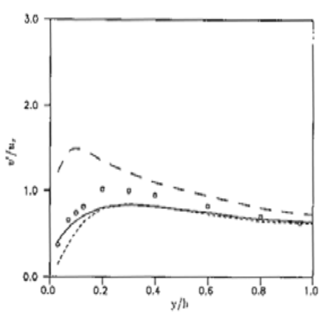
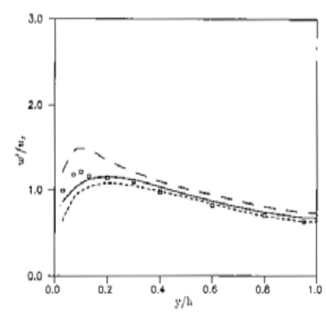


Vertical turbulence intensity



Lateral turbulence intensity

Duct Flows

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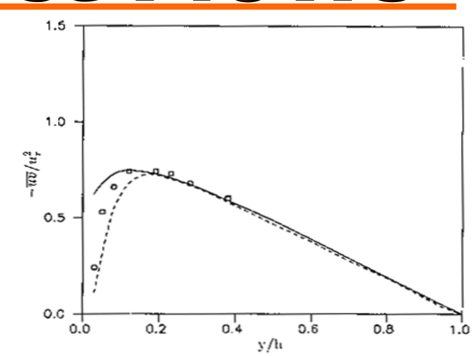

Turbulence shear stress

Backward Facing Step Flows

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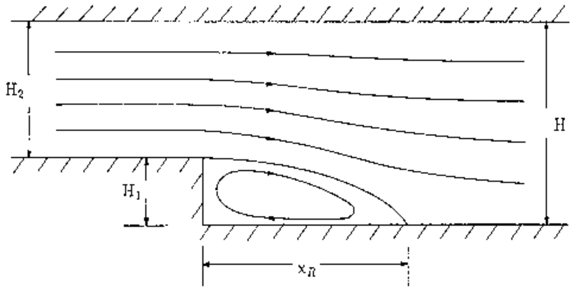

Comparison are with the experimental data of Kim et al. (1978) and simulation of Srinivasan et al. (1983)

Backward Facing Step Flows

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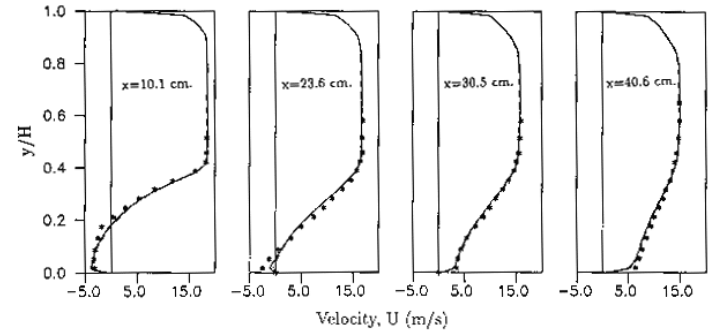

Mean Velocity Profiles

Backward Facing Step Flows

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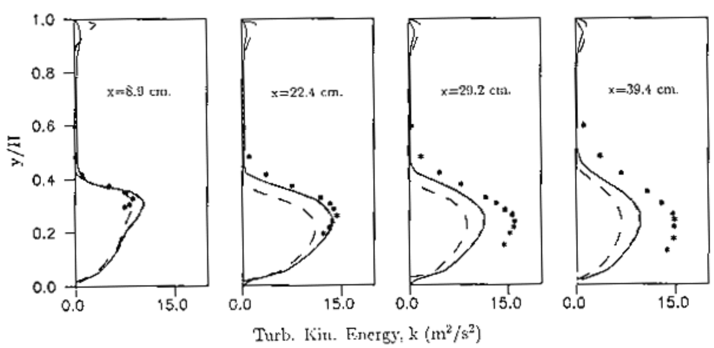

Turbulence Kinetic Energy Profiles

Backward Facing Step Flows

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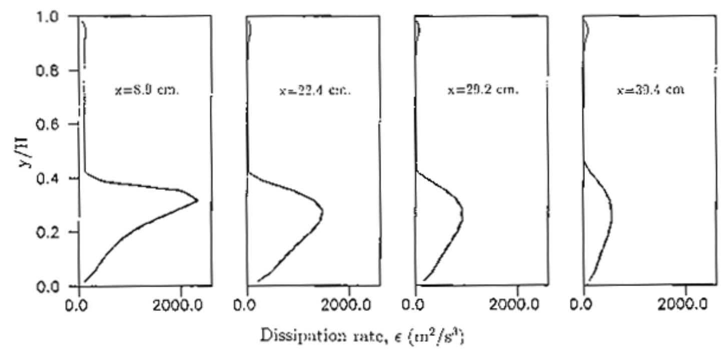

Turbulence Dissipation Rate Profiles

Backward Facing Step Flows

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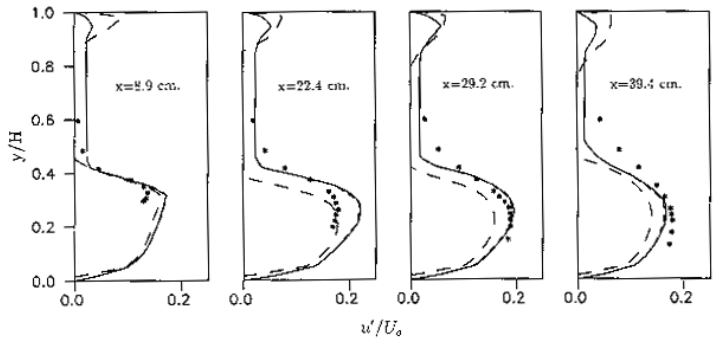

Axial Turbulence Intensity Profiles

Backward Facing Step Flows

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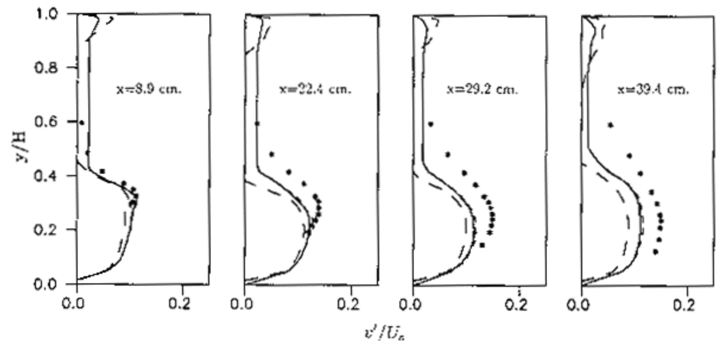

Vertical Turbulence Intensity Profiles

Backward Facing Step Flows

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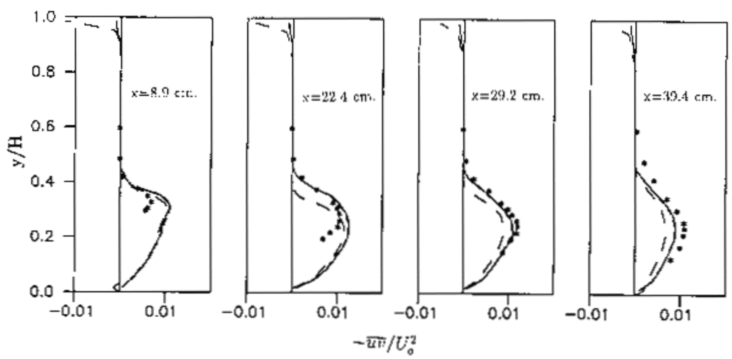

Turbulence Shear Stress Profiles

Axisymmetric Pipe Expansion Flows

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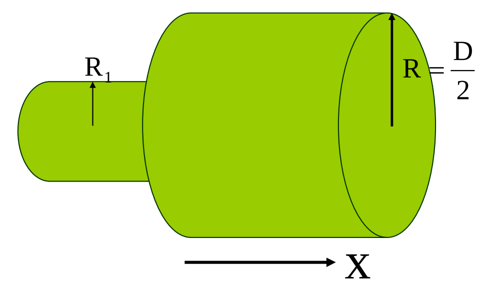

Comparison are with the experimental data of Juniua et al. (1982) and Chaturvedi (1963) and simulation of Srinivasan et al. (1983)

Axisymmetic Pipe Expansion Flows

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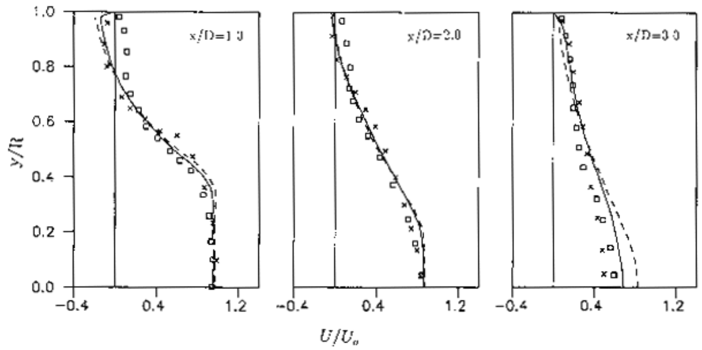

Mean Velocity Profiles

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Axisymmetic Pipe Expansion Flows

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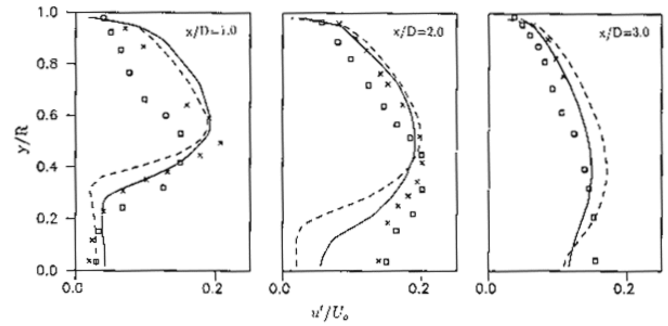

Axial Turbulence Intensity Profiles

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Axisymmetic Pipe Expansion Flows

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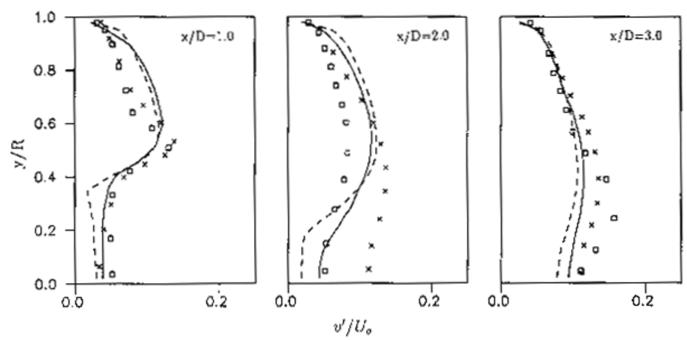

Vertical Turbulence Intensity Profiles

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Axisymmetic Pipe Expansion Flows

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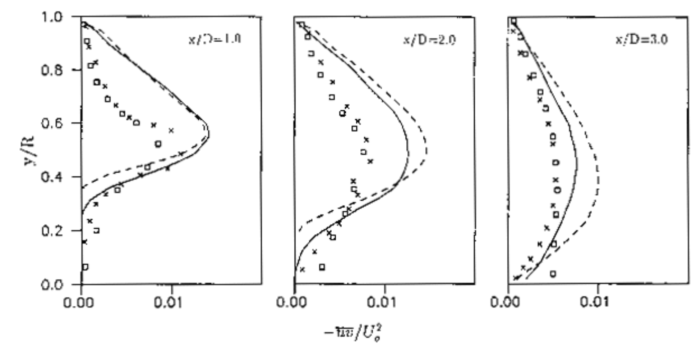

Turbulence Shear Stress Profiles

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Conclusions

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- Thermodynamics provides guidelines for turbulence modeling.
- Rate-dependent model provides some improvements over the existing two-equation models.
- The rate-dependent model could be extended to more complex turbulent multiphase flows.