Turbulent Jet Flows

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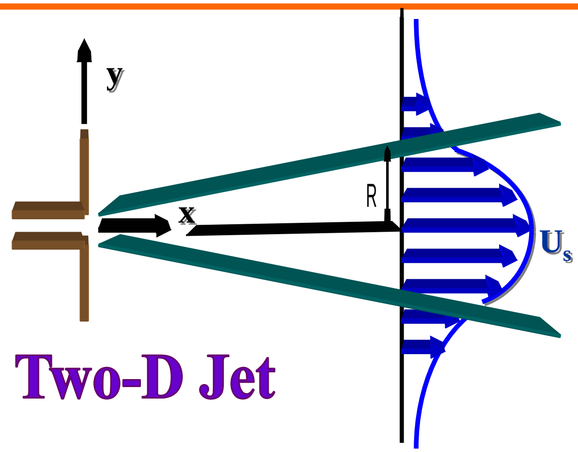
Objectives

- Turbulent Jet Flows
- Momentum Integral
- Similarity Variable
- Eddy Viscosity Model
- Similarity Solution
- Thermal Plume



Equation of Motion

$$U\frac{\partial U}{\partial x}+V\frac{\partial U}{\partial y}+\frac{\partial}{\partial y}\overline{u'v'}=0$$

Momentum Integral

$$\frac{d}{dx}\int_{-\infty}^{+\infty}U^2dy=0$$

$$\rho\int_{-\infty}^{+\infty}U^2dy=M=\text{Jet Momentum}$$

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Continuity

$$\frac{\partial U}{\partial x}+\frac{\partial V}{\partial y}=0$$

Stream Function

$$U=\frac{\partial \psi}{\partial y}$$

$$V=-\frac{\partial \psi}{\partial x}$$

$$\xi=\frac{y}{\ell}$$

Self Similar Solutions

$$U=U_s f(\xi)$$

$$-\overline{u'v'}=U_s^2 g(\xi)$$

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Similarity Variables

$$U_s=cx^m \qquad \ell=Dx^n$$

Momentum Equation → $2m-1=2m-n$ → $n=1$

Momentum Integral → $2m+n=0$ → $m=-\frac{1}{2}$

Then

$$U_s=Cx^{-\frac{1}{2}} \qquad \ell=Dx$$

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$$\xi=\frac{y}{\ell}=\frac{y}{Dx} \qquad \frac{\partial \xi}{\partial x}=-\frac{y}{Dx^2}=-\frac{\xi}{x} \qquad \frac{\partial \xi}{\partial y}=\frac{1}{\ell}=\frac{1}{Dx}$$

Stream Function

$$\psi=CDx^{\frac{1}{2}}F(\xi)$$

Mean U-Velocity

$$U=Cx^{-\frac{1}{2}}F'(\xi)$$

Turbulent Shear Stress

$$-\overline{u'v'}=C^2x^{-1}g(\xi)$$

Mean V-Velocity

$$V=-\frac{\partial \psi}{\partial x}=-CDx^{-\frac{1}{2}}\left(\frac{1}{2}F-\xi F'\right)$$

Eddy Viscosity

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Momentum Equation

$$F'^2+FF''=-\frac{2}{D}g'$$

Turbulence Shear Stress

$$-\overline{u'v'}=\nu_T\frac{\partial U}{\partial y}$$

Eddy viscosity

$$\frac{\nu_T}{U_s\ell}=\frac{1}{R_T}=\frac{g}{F''}$$

$$g=\frac{1}{R_T}F''$$

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$$(FF')' + F''' = 0 \qquad FF' + F'' = c_1$$

Boundary Conditions

$$F'(0) = 1 \qquad F'(\infty) = 0 \qquad F(0) = 0$$

Mean Velocity ⇒ $F = \sqrt{2}\tanh\frac{\xi}{\sqrt{2}}$

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Mean Velocity ⇒ $f = F' = \frac{1}{\cosh^2\frac{\xi}{\sqrt{2}}} = \sec h^2\frac{\xi}{\sqrt{2}}$

Similarity Variables

$$\frac{U_s}{U_j} = 2.7\left(\frac{d}{x}\right)^{\frac{1}{2}} \qquad \ell = 0.078x$$

Momentum Integral ⇒ $\int_{-\infty}^{+\infty} U^2 dy = U_j^2 d$

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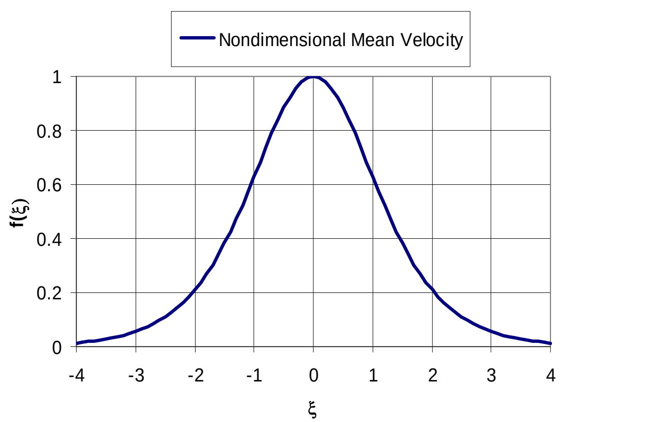

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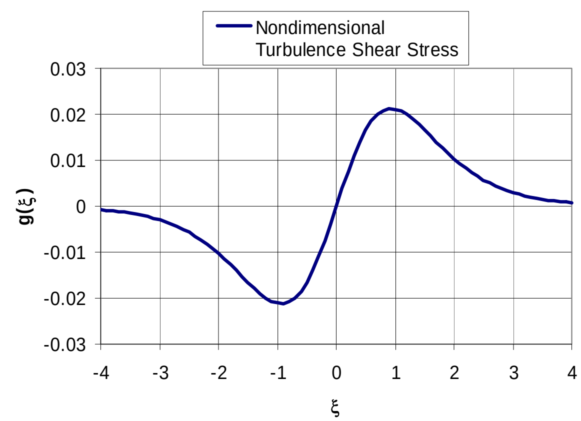

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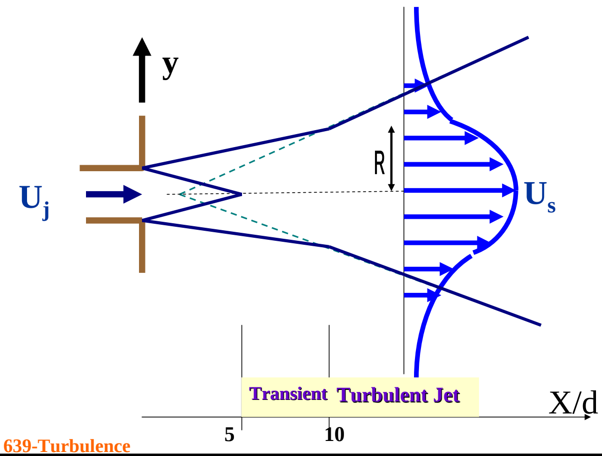

Turbulent Thermal Plume

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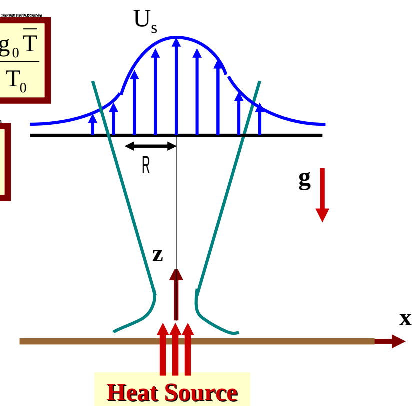
$$U\frac{\partial W}{\partial x}+W\frac{\partial W}{\partial z}+\frac{\partial}{\partial x}\overline{u'w'}=\frac{g_0\bar{T}}{T_0}$$

$$U\frac{\partial \bar{T}}{\partial x}+W\frac{\partial \bar{T}}{\partial z}+\frac{\partial}{\partial x}\overline{T'u'}=0$$

$$\frac{\partial U}{\partial x}+\frac{\partial W}{\partial z}=0$$

$$\int_{-\infty}^{+\infty}\bar{T}W\,dx=\text{const}=\frac{q}{\rho c_p}$$



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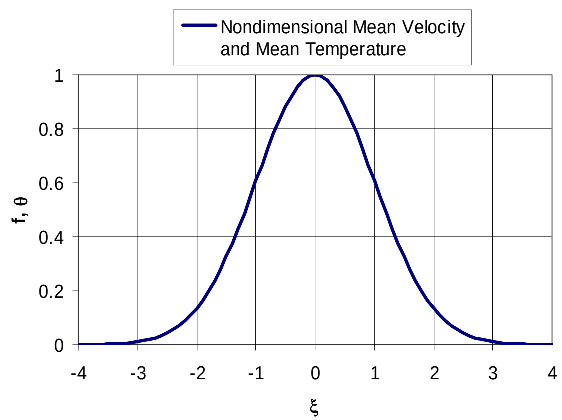

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Concluding Remarks

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