

PHENOMENOLOGICAL MODELS

Goodarz Ahmadi

Department of Mechanical and Aeronautical Engineering
Clarkson University
Potsdam, NY 13699-5727

Outline

- Reynolds Equation
- Eddy Viscosity Models
- Mixing Length Model
- Near Wall Flows

Navier-Stokes

$$\rho \left(\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} \right) = -\frac{\partial p}{\partial x_i} + \mu \frac{\partial^2 u_i}{\partial x_j \partial x_j}$$

$$\frac{\partial u_i}{\partial x_i} = 0$$

Turbulence

$$\mathbf{u} = \mathbf{U} + \mathbf{u}' \quad U_i = \bar{u}_i \quad \bar{u}'_i = 0$$

$$p = P + p' \quad P = \bar{p} \quad \bar{p}' = 0$$

Time Averaging $\Rightarrow \bar{u}_i = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{t_0}^{t+T} u_i dt$

Ensemble Averaging $\Rightarrow \langle u_i \rangle = \int_{-\infty}^{+\infty} u_i f(\mathbf{u}) d\mathbf{u}$

Ergodicity $\Rightarrow \bar{u}_i = \langle u_i \rangle = U_i$

Averaging

Properties

$$\overline{u'_i} = 0$$

$$\overline{p'} = 0$$

$$\overline{u'_i u'_j} \neq 0$$

$$\overline{p' u'_i} \neq 0$$

$$\overline{u'_i u'_j u'_k} \neq 0$$

$$\overline{U_i u'_j} = U_i \overline{u'_j} = 0$$

$$\frac{\partial \overline{u'_i}}{\partial x_j} = 0$$

Phenomenological Models For Turbulence

Reynolds Equation

$$\rho \left(\frac{\partial U_i}{\partial t} + U_j \frac{\partial U_i}{\partial x_j} \right) = -\frac{\partial P}{\partial x_i} + \mu \frac{\partial^2 U_i}{\partial x_j \partial x_j} - \rho \frac{\partial \overline{u'_i u'_j}}{\partial x_j}$$



$$\frac{\partial U_i}{\partial x_i} = 0$$

Turbulence Stress

$$\tau_{ij}^T = -\rho \overline{u'_i u'_j} = \tau_{ji}^T$$

Phenomenological Models For Turbulence

Reynolds Equation

$$\rho \left(\frac{\partial U_i}{\partial t} + U_j \frac{\partial U_i}{\partial x_j} \right) = \frac{\partial}{\partial x_j} \left[-P \delta_{ij} + \mu \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) - \rho \overline{u'_i u'_j} \right]$$

Turbulence Stress

Reynolds Stress

$$\boldsymbol{\tau}^T = \begin{pmatrix} -\rho \overline{u'^2} & -\rho \overline{u'v'} & -\rho \overline{u'w'} \\ -\rho \overline{u'v'} & -\rho \overline{v'^2} & -\rho \overline{v'w'} \\ -\rho \overline{u'w'} & -\rho \overline{v'w'} & -\rho \overline{w'^2} \end{pmatrix}$$

Phenomenological Models For Turbulence

Boussineq Eddy Viscosity Model

$$\tau_{ij}^T = -\frac{\rho \overline{u'_k u'_k}}{3} \delta_{ij} + \mu_T \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right)$$

Eddy Viscosity

$$\mu_T = \rho \nu_T$$

$$\tau_{12}^T = \tau^T = \rho \nu_T \frac{\partial U}{\partial y}$$

Prandtl Mixing Length Model Clarkson University

Prandtl Assumption



Ludwig Prandtl

$$\overline{(u'^2)^2} \sim \overline{(v'^2)^2} \sim \ell \frac{dU}{dy}$$

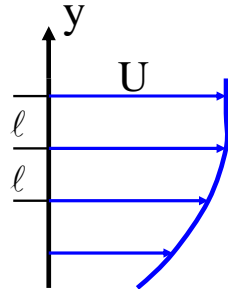
$$\tau^T = -\rho \overline{u'v'}$$

Mixing Length

$$\tau^T = \rho \ell^2 \left| \frac{\partial U}{\partial y} \right| \frac{\partial U}{\partial y}$$

Eddy Viscosity

$$\nu_T = \ell^2 \left| \frac{\partial U}{\partial y} \right|$$



$$\ell = \kappa \left| \frac{dU}{dy} \right| \frac{d^2U}{dy^2}$$

von Karman

ME 639-Turbulence

G. Ahmadi

Velocity Near a Wall Clarkson University

Inertial Sublayer

Shear Velocity

$$u^* = \sqrt{\frac{\tau_0}{\rho}}$$

Turbulence Scales

$$\ell = \kappa y$$

$$\kappa = 0.4$$

von Karman constant

Inertial Sublayer



von Karman

ME 639-Turbulence

G. Ahmadi

Inertial Sublayer Clarkson University

Turbulent Stress=Wall Shear Stress

$$\tau_0 = \rho \kappa^2 y^2 \left(\frac{\partial U}{\partial y} \right)^2$$

$$\frac{dU}{dy} = \frac{u^*}{\kappa y}$$

$$\frac{U}{u^*} = U^+ = \frac{1}{\kappa} \ln y + c$$

Wall Units

$$U^+ = \frac{1}{\kappa} \ln y^+ + B$$

$$y^+ = \frac{u^* y}{\nu}$$

$$B \approx 5$$

$$30 < y^+ \leq 300$$

ME 639-Turbulence

G. Ahmadi

Viscous Sublayer Clarkson University

Turbulent stress is negligible

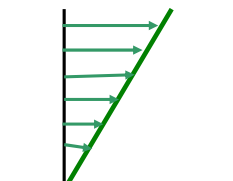
$$0 < y^+ \leq 5$$

$$\tau_0 = \mu \frac{dU}{dy}$$

$$u^{*2} = \nu \frac{dU}{dy}$$

$$\frac{dU^+}{dy^+} = 1$$

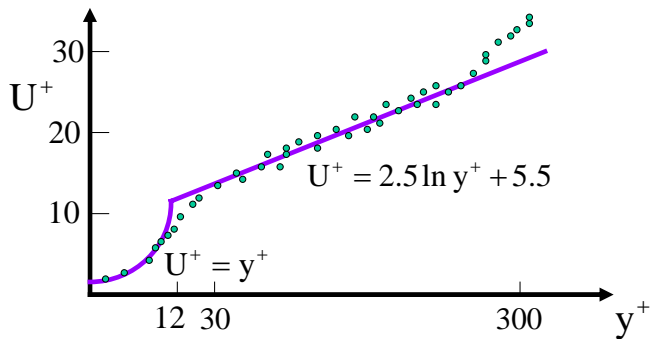
$$U^+ = y^+$$



ME 639-Turbulence

G. Ahmadi

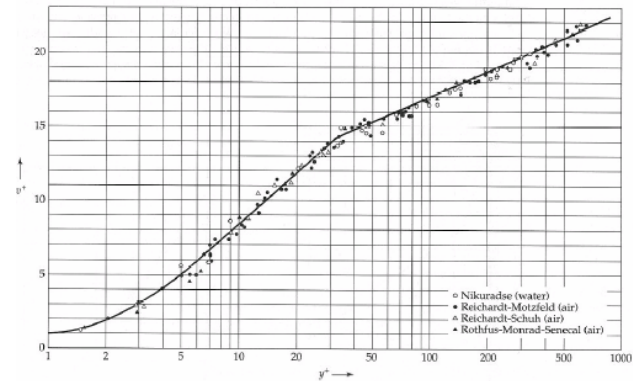
Velocity Near a Wall Clarkson University



ME 639-Turbulence

G. Ahmadi

Velocity Near a Wall Clarkson University



ME 639-Turbulence

G. Ahmadi

Phenomenological Models Clarkson University

Concluding Remarks

- Reynolds Equation
- Eddy Viscosity Models
- Mixing Length Model
- Near Wall Flows

ME 639-Turbulence

G. Ahmadi