STABILITY OF VISCOUS FLOW

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Outline

- Linear Stability
- Disturbed Motion
- Orr-Sommerfeld Equation
- Stability Conditions
- Squire Theorem

LINEAR STABILITY ANALYSIS

Navier-Stokes Equation

$$\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v} = -\nabla p + \frac{1}{\mathrm{Re}}\nabla^2\mathbf{v}$$

$$\nabla\cdot\mathbf{v} = 0$$

Basic flow (Steady) $V(\mathbf{x}),\ P$

Basic Flow (Steady)

$$\mathbf{V}\cdot\nabla\mathbf{V} = -\nabla\mathbf{P} + \frac{1}{\mathrm{Re}}\nabla^2\mathbf{V}$$

$$\nabla\cdot\mathbf{V} = 0$$

Disturbed Flow (unsteady)

$$\mathbf{v} = \mathbf{V} + \mathbf{v}'$$

$$|v'| << |V|$$

$$p = P + p'$$

Governing Equations for Perturbed Motion

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$$\frac{\partial \mathbf{v}'}{\partial t} + \mathbf{V}\cdot\nabla\mathbf{v}' + \mathbf{v}'\cdot\nabla\mathbf{V} = -\nabla p' + \frac{1}{Re}\nabla^2\mathbf{v}'$$

$$\nabla\cdot\mathbf{v}' = 0$$

Stability of Two-Dimensional Parallel Flows

Basic flow $\mathbf{V} = U(y)\mathbf{i}$ $P = P(x, y)$

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Governing Equations for Perturbed Motion

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$$\frac{\partial u'}{\partial t} + U\frac{\partial u'}{\partial x} + v'\frac{\partial U}{\partial y} = -\frac{\partial P'}{\partial x} + \frac{1}{Re}\nabla^2 u'$$

$$\frac{\partial v'}{\partial t} + U\frac{\partial v'}{\partial x} = -\frac{\partial P'}{\partial y} + \frac{1}{Re}\nabla^2 u'$$

$$\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} = 0$$

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Governing Equations for Perturbed Motion

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Stream Function $u' = \frac{\partial \psi}{\partial y}$ $v' = -\frac{\partial \psi}{\partial x}$

$$\frac{\partial}{\partial t}\nabla^2\psi + U\frac{\partial}{\partial x}\nabla^2\psi - \frac{\partial \psi}{\partial x}U'' = \frac{1}{Re}\nabla^4\psi$$

Propagating Wave Solution

$$\psi = \varphi(y)e^{i\alpha(x-ct)}$$

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Governing Equations for Perturbed Motion

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Complex Speed $c = c_r + ic_i$

Wave Number = α

$c_i > 0$ ⇨ **Instability**

$c_i < 0$ ⇨ **Stability**

Orr-Sommerfeld Equation

$$(U-c)(\varphi'' - \alpha^2\varphi) - U''\varphi = -\frac{i}{\alpha Re}(\varphi'''' - 2\alpha^2\varphi'' + \alpha^4\varphi)$$

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Boundary Conditions

$$y = 0\,,\ u' = v' = 0 \ \text{ or } \ \varphi(0) = \varphi'(0) = 0$$

$$y = \infty\,,\ u' = v' = 0 \ \text{ or } \ \varphi(\infty) = \varphi'(\infty) = 0$$

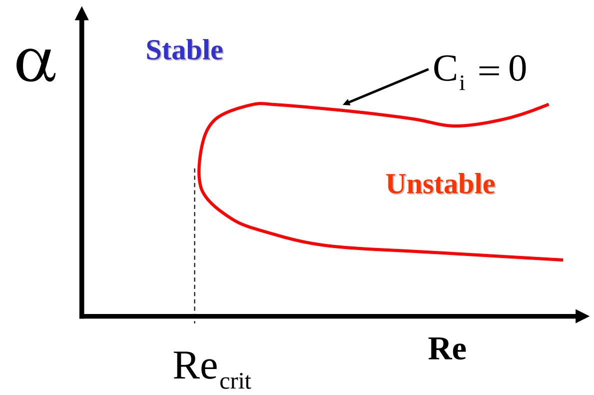
Critical Re

$$\mathrm{Re}_{crit} = \left.\frac{U_\infty \delta^*}{\nu}\right|_{crit} = 520$$

Experimental Re_{crit}=950-1700

$$\mathrm{Re}_x = \frac{U_\infty x}{\nu} \approx 3.2\times 10^5 \sim 10^6$$

Stability Conditions



Squire Theorem

Two-dimensional disturbances are more critical in comparison to the three dimensional disturbances for two-dimensional flows.

Frictionless Stability Analysis

$$(U - c)(\varphi'' - \alpha^2\varphi) - U''\varphi = 0$$

$$\varphi(0) = \varphi(\infty) = 0$$

Stability Theorem (Rayleigh, Tollmien)

The boundary layer velocity profiles that possess a point of inflexion are unstable.

Frictionless Stability Analysis

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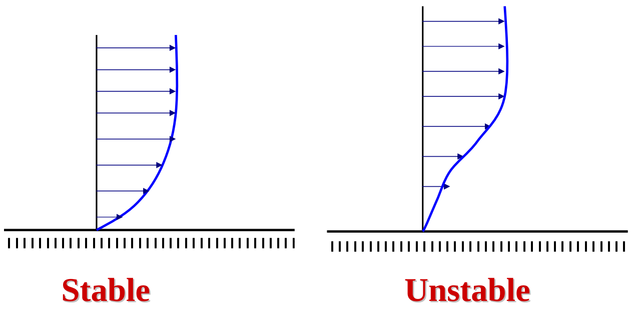

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Concluding Remarks

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- Orr-Sommerfeld Equation
- Stability Conditions
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