# ME 639 - Turbulence

# FUNDAMENTALS

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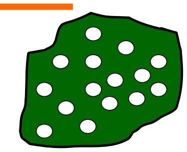
# FUNDAMENTALS

**Continuum Assumption**

**Fluid is a Continuum**



$3 \times 10^{19}$ molecules in 1 cm$^3$

**Density:**

$$\rho = \lim_{L \to 0} \frac{\sum m_i}{V}$$

**Velocity (mass-averaged)**

$$\mathbf{v} = \lim_{L \to 0} \frac{\sum m_i \mathbf{v}_i^{(k)}}{\sum m_i}$$

**Molar Averaged Velocity**

$$\mathbf{V}^{(k)} = \lim_{L \to 0} \frac{\sum \mathbf{v}_i^{(k)}}{n^{(k)}}$$



# Properties

**Fluctuation Velocity**

$$\mathbf{v}_i' = \mathbf{v}_i - \mathbf{v}$$

**Internal Energy Density**

$$e = \lim_{L \to 0} \frac{\sum m_i \frac{1}{2} \overline{\mathbf{v}'_i \cdot \mathbf{v}'_i}}{\sum m_i}$$

**Temperature**

$$\frac{3}{2} kT = \frac{1}{2} m \overline{\mathbf{u}'^2}$$



# Thermodynamics

- Thermodynamic properties (temperature, entropy,
- internal energy, enthalpy, etc.) are related.
- For a thermodynamic state, all properties are
- specified.
- A process constitutes a change in state.
- Reversible process is a sequence of
- thermodynamical state.
- Extensive properties are proportional to the mass
- of the system.
- Intensive properties are independent of the mass
- of the system.

## Entropy

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**Entropy measures the irreversibility of a process**

$$ds = \frac{dQ}{T}$$

**(For reversible processes)**

**Statistical Mechanics (Boltzmann)**

$$s = k \ln f$$



## Basic Thermodynamical Equations

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$$Tds = de + pd\vartheta$$

$$de = Tds - pd\vartheta = Tds + \frac{p}{\rho^2} d\rho$$

$$T = \left.\frac{\partial e}{\partial s}\right|_{\vartheta}$$

$$p = -\left.\frac{\partial e}{\partial \vartheta}\right|_{s} = \rho^2 \left.\frac{\partial e}{\partial \rho}\right|_{s}$$



## Helmholtz Free Energy Function

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$$\psi = e - Ts \qquad \psi = \psi(T, \rho)$$

$$s = -\left.\frac{\partial \psi}{\partial T}\right|_{p} \qquad p = \rho^2 \left.\frac{\partial \psi}{\partial \rho}\right|_{T}$$

**Enthalpy**

$$h = e + \frac{p}{\rho}$$

**Isothermal Compressibility**

$$\alpha = \frac{1}{\rho} \left.\frac{\partial \rho}{\partial P}\right|_{T}$$

**Bulk Expansion**

$$\beta = \frac{1}{\rho} \left.\frac{\partial \rho}{\partial T}\right|_{p}$$



## Ideal Gas

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$$p = \rho R T \qquad h = e + RT$$

$$c_P = \left.\frac{\partial h}{\partial T}\right|_{P} = c_V + R \qquad c_V = \left.\frac{\partial e}{\partial T}\right|_{P} \qquad \gamma = \frac{c_P}{c_V}$$

**Incompressible Substance**

$$c_P = c_V \qquad \gamma = 1$$

**Compressibility Factor**

$$Z = \frac{p}{\rho R T}$$