ME 639 SIMULATION OF CHAOS, PROJECT-1 Spring 2024

Select a nonlinear deterministic dynamical system for detailed analysis.

1. Study the equilibrium states and periodic orbit solutions. (Analyze the stability of the equilibrium states.)
2. Perform numerical experiments for the cases that which periodic and chaotic solution exists.
3. Plot sample response time histories for various conditions.
4. Plot sample responses in the phase plane.
5. Plot the Poincare map for the responses.
6. Evaluate the response statistics, including mean, mean square, various moments, auto-correlation, power spectrum, and cross-correlation.
7. Repeat parts (1-6) for the case that a white noise excitation is present.
8. * Evaluate the orthogonal basis for the Karhunen-Loeve (K-L) expansion.
9. * Compare the statistical properties of the K-L expansion with the original field.
10. * Apply a moment/pdf closure and compare the accuracy of the results with the direct numerical simulation results.
11. * Develop an active control scheme for control of the chaotic response.

(* items are for extra credits.)

Examples of Possible Nonlinear Systems:

- Lorentz Model

$$\begin{cases} \dfrac{dx}{dt} = \sigma(y - x) \\ \dfrac{dy}{dt} = rx - y - xz \\ \dfrac{dz}{dt} = -bz + xy \end{cases} \qquad (\sigma,\ r,\ b \ \text{are consants.})$$

- Double diffusive Convection

$$\begin{cases} \dfrac{dx}{dt} = \sigma(-x + r_T y - r_s z) \\ \dfrac{dy}{dt} = -y + x - xz \\ \dfrac{dz}{dt} = a(-z + xy) \end{cases} \qquad \begin{aligned} \frac{du}{dt} &= -bu + x(1 - v) \\ \frac{dv}{dt} &= a(-bv + xu) \end{aligned}$$

- Van der Pol Equation

$$\ddot{x} + (x^2 - 1)\dot{x} + x^3 = B\cos\omega t$$

- Pendulum

$$\ddot{x} + 2\xi\dot{x} + \alpha\sin x = \gamma\cos\omega t$$

- Double well Potentials

$$\ddot{x} + \beta\dot{x} - \frac{1}{2}x(1 - x^2) = \gamma\cos\omega t$$

- Vibrating Pendulum

$$\ddot{x} + 2\xi\dot{x} + (1 + \gamma\sin\omega t)\sin x = 0$$

Due date: The project reports should include an abstract, introduction, results, figures with captions, a discussion of the results, conclusions, and references. The PDF copy of the report and all programs should be uploaded to Moodle by **February 22, 2024**.