Project 1

Consider the following nonlinear Langevin equation,

$$\ddot{\mathbf{x}} + 0.02\dot{\mathbf{x}} + \mathbf{x} + \boldsymbol{\varepsilon}\mathbf{x}^3 = \mathbf{f(t)}, \quad \mathbf{x(0)} = \mathbf{0}, \quad \dot{\mathbf{x}}\mathbf{(0)} = \mathbf{0} \tag{1}$$

For this system,

a) Develop a computer program for evaluating **x**(t).

b) For **f(**t) = sint, **ε** = 0, determine the exact solution for **x**(t). Evaluate the numerical solution of (1) and compare the result with the exact solution in a figure.

c) Evaluate **x**(t) numerically for **ε** = 0.5 and 1, plot the results, and discuss the effect of **ε**.

d) Assume f(t) is a white noise process with $S_o = 55.44 cm^2/s^3$. Simulate the white noise process numerically. For an ensemble of 200 samples, evaluate $\sigma_x^2(t)$ for a time duration of 20-sec use **ε** = 0 and **ε** = 0.1. For **ε** = 0 compare the simulation results with the exact solution.

e) For **ε** = 0.1, use the method of equivalent linearization and evaluate the stationary limit of E{$\mathbf{x^2}$}. Compare the results with the simulation results.

f) Set up the Fokker-Planck equation and determine its exact stationary solution. Compare the results for **ε** = 0 and **ε** = 0.1, with the simulation results.

g) Setup the moment of Fokker-Planck equations and evaluate E{$\mathbf{x^2}$(t)}. Compare the results with the simulation results.

h) (Extra Points) For **ε** = 0, evaluate the Karhunen - Loeve orthogonal basis. Consider a three-term expansion and evaluate the variance of the response when one, two, or three terms are considered. Compare the results with the exact and simulation results. Also, compare some sample solutions.

Project 2

Apply what you learned in this course to your thesis. For example, develop a stochastic model, stochastic estimation, Neural Network, and machine learning estimation for some data in your thesis or environmental, biological, or industrial data.

Report and Due Date: The final report should include hard copies of the technical report, figures, and a discussion of the results. An electronic copy, including the report and all the programs, should be submitted to Moodle. The due date for the project is **December 1, 2023**.

Guideline for Technical Report Writing

First page:
Title of the project

Names of the authors and their affiliations

Abstract

Describe what is in the report, the key points, and significant findings.

Second Page
Introduction

Introduce the topic and give the background. Review the related works. In the literature survey, point to the gaps in the literature and set the stage for this work. Also, give a summary of the main findings.

Technical Report

Formulation or Experimental setup
Experimental procedure
Results and Discussion
Note that all equations should be numbered consecutively. In addition, all assumptions should be stated clearly.
Note that all figures need to be numbered consecutively, and each figure should be discussed.

Conclusions

State the conclusions of the study.

References

All references should be listed. All listed references should be referred to in the text.

Make sure to keep the same style in the references. References are given either by year or are numbered. For example,

Smith, J.D. (2000). Title of the paper. Journal of Probabilistic Mechanics, Vol. 4, pp. 345-356.

1. Smith, J.D., Title of the paper. Journal of Probabilistic Mechanics, Vol. 4, pp. 345-356 (2000).