Turbulence Deposition

During turbulent fluid motions, particles are transported by the turbulence eddies and the Brownian diffusion. Thus, the particle flux is given by

$$J = (D + \nu^T)\frac{dC}{dy} \tag{1}$$

where C is the average concentration and ν^T is the turbulent eddy diffusivity. According to Lin et al.,

$$\frac{\nu^T}{\nu} = \begin{cases} \left(\dfrac{y^+}{14.5}\right)^3 & 0 \le y^+ \le 5 \\ \dfrac{y^+}{5} - 0.959 & 5 \le y^+ \le 30 \end{cases} \tag{2}$$

where

$$y^+ = \frac{yu^*}{\nu}, \qquad u^* = \sqrt{\frac{\tau_0}{\rho^f}}. \tag{3}$$

Assuming that the flux is a constant (or linearly varying) near the wall,

$$\frac{dC}{dy} = \frac{J}{D + \nu^T}. \tag{4}$$

Subject to appropriate boundary conditions, the concentration and the flux to the wall may be determined. This approach was first introduced by Friedlander and Johnstone (1959). Since then many authors used a number of modified boundary conditions. Here the empirical model described by Wood (1981) is outlined.

Turbulent Diffusion (Wood, J. Aerosol Science, 12, pp 276-281, 1981)

For turbulent flow near a wall, using the shear velocity $u^* = \sqrt{\frac{\tau_0}{\rho^f}}$ and ν, a wall unit $\frac{\nu}{u^*}$ may be defined. All quantities may be non-dimensionalized with the aid of u^* and $\frac{\nu}{u^*}$. e.g., the fluid mean velocity $u^+ = \frac{u}{u^*}$ and the particle relaxation time

$$\tau^+ = \frac{u^{*2}\tau}{\nu} = \frac{\rho^p d^2 u^{*2}}{18\rho^f \nu^2} = \frac{1}{18}(\frac{\rho^p}{\rho^f})\,\mathrm{Re}_d^{*2}, \tag{5}$$

where the Reynolds number based on shear velocity is defined as, $\mathrm{Re}_d^* = \frac{u^* d}{\nu}$. The non-dimensional deposition velocity is defined as

$$u_D^+ = \frac{u_D}{u^*} = \frac{J}{c_0 u^*} = \frac{D + \nu^T}{\nu}\frac{dc^+}{dy^+}. \tag{6}$$

For very small particles with $\tau^+ \le 1$, Brownian diffusion becomes significant and deposition is affected by a combination of Brownian and eddy diffusion. For the Brownian regime,

$$u_D^+ \approx \frac{3\sqrt{3}}{29\pi} s_c^{-2/3} = 0.057 s_c^{-2/3} \tag{7}$$

where s_c is the Schmidt number. For larger particles up to $\tau^+ \approx 10$, Equation (7) must be modified for the eddy diffusion-impaction regime:

$$u_D^+ = 0.057 s_c^{-2/3} + 4.5\times 10^{-4}\tau^{+2}, \tag{85}$$

The second term becomes dominant in the eddy-diffusion-impaction regimes for $1 < \tau^+ < 10$.

In the particle inertia moderated regime,

$$u_D^+ = 0.13 \qquad 17 \le \tau^+ \le 200 \tag{86}$$

For $\tau^+ \ge 265$, it was found empirically

$$u_D^+ \approx \frac{2.6}{\sqrt{\tau^+}}(1 - \frac{50}{\tau^+}). \tag{87}$$

Figure 1 shows the variation of the nondimensional deposition velocity with nondimensional relaxation time for a turbulent flow with shear velocity of 0.3 m/s. The particles are assumed to have a density ratio of 2000 in this example.

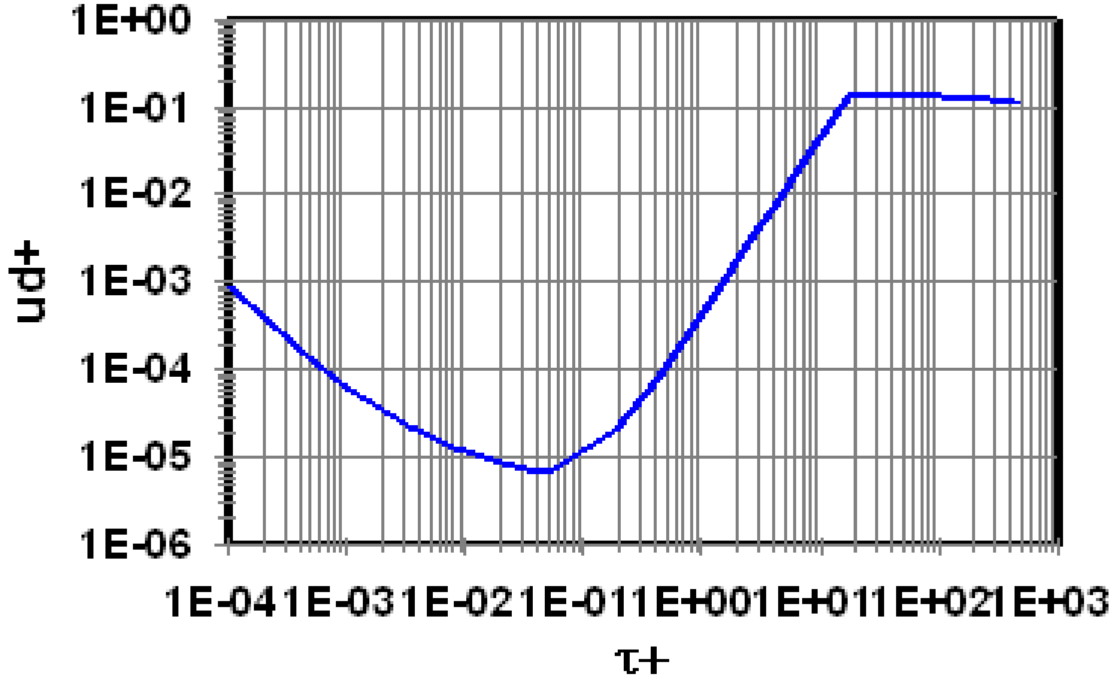

Figure 1. Variation of the nondimensional deposition velocity with nondimensional relaxation time for $u^* = 0.3$ m/s and S=2000.

Deposition on Rough Walls

Wall roughness k could significantly affect the deposition rate in turbulent streams. Wood suggested

$$u_D^+ = \frac{1}{I_B + I_s}, \; I_B = 24.2 \text{ for } 0.45k^+ < 5, \tag{88}$$

where

$$I_s = 14.5 S_c^{2/3} [\frac{1}{6} \ln \frac{(1+\phi)^2}{1-\phi+\phi^2} + \frac{1}{\sqrt{3}} \tan^{-1} \frac{2\phi - 1}{\sqrt{3}} - \frac{1}{6} \ln \frac{(1+\phi_k)^2}{1-\phi_k+\phi_k^2} - \frac{1}{\sqrt{3}} \tan^{-1} \frac{2\phi_k - 1}{\sqrt{3}}]$$
$$\text{for } 0.45k^+ < 5 \tag{89}$$

Here

$$k^+ = \frac{ku^*}{\nu}, \; \phi = \frac{1}{2.9} s_c^{1/3}, \; \phi_k = \frac{k^+}{32.2} s_c^{1/3}, \; s_c = \frac{\nu}{D}. \tag{90}$$

Equations (88) and (89) may be approximated as

$$u_D^+ = [\frac{1524}{0.45k^t(0.9k^+ - 5/\phi)} - 6.3]^{-1} \text{ for } \phi_k >> 1, \, (k^+ \geq 1) \tag{91}$$

$$u_D^+ = [24.2 + 14.5 s_c^{2/3} (\frac{2\pi}{3\sqrt{3}} - \frac{k^+}{27.6} s_c^{1/3})]^{-1} \text{ for } \phi_k >> 1. \tag{92}$$

For $k^+ \to 0$, equation (92) reduces to (84).

For the highly rough case $5 < 0.45k^+ << 30$, $I_s = 0$

and

$$I_B = 5 \ln(\frac{25.2}{0.45k^+ - 4.8}), \tag{93}$$

or

$$u_D^+ = \frac{0.2}{\ln(\frac{25.2}{0.45k^+ - 4.8})}. \tag{94}$$

Figure 2 shows the variation of the nondimensional deposition velocity with nondimensional relaxation time for smooth and rough surfaces as predicted by Equation (89) (Wood's formula.). It is seen that as the roughness coefficient increases the deposition rate increases significantly. For a large value of surface roughness, the deposition rate becomes roughly a constant.

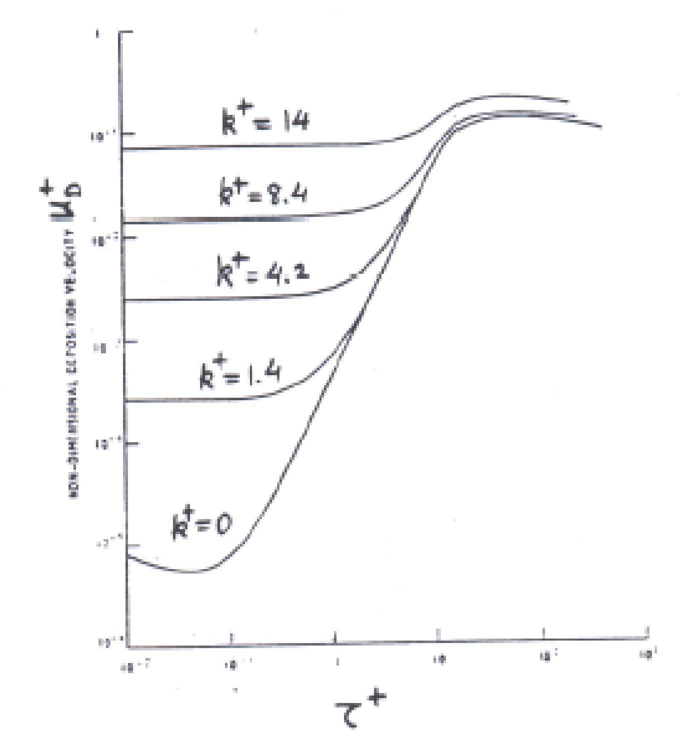

Figure 2. Variation of the nondimensional deposition velocity with nondimensional relaxation time for smooth and rough surfaces as predicted by Wood's formula.

Using a sublayer model, Fan and Ahmadi developed a semi-empirical model for the deposition rate on smooth and rough surfaces. Accordingly,

$$
u_d^+ = \begin{cases} 0.084\,Sc^{-2/3} + \dfrac{1}{2}\left[\dfrac{\left(0.64k^+ + \dfrac{d^+}{2}\right)^2 + \dfrac{\tau_p^{+2} g^+ L_1^+}{0.01085(1+\tau_p^{+2} L_1^+)}}{3.42 + \dfrac{\tau_p^{+2} g^+ L_1^+}{0.01085(1+\tau_p^{+2} L_1^+)}}\right]^{1/\left(1+\tau_p^{+2} L_1^+\right)} & \\ \quad \times\left[1+8e^{-(\tau_p^+-10)^2/32}\right]\dfrac{0.037}{1-\tau_p^{+2} L_1^+\left(1+\dfrac{g^+}{0.037}\right)} & \text{if } u_d^+ < 0.14 \\ 0.14 & \text{otherwise} \end{cases} \tag{95}
$$

Here d^+ is the particle diameter in wall units, τ_p^+ is the particle relaxation time in wall units, $g^+ = \nu g / u^{*3}$, and $L_1^+ = 3.08 / Sd^+$. The predictions of Equation (95) are shown in Figure 3 and are compared with that of Wood's equation for smooth wall. It is seen that the model predictions of Fan-Ahmadi are comparable with that of Wood for a smooth surface. For rough surfaces the deposition rate increases sharply as the surface roughness increases. Fan Ahmadi (1993) showed that the empirical model prediction for rough surfaces are in reasonable agreement with the experimental data.

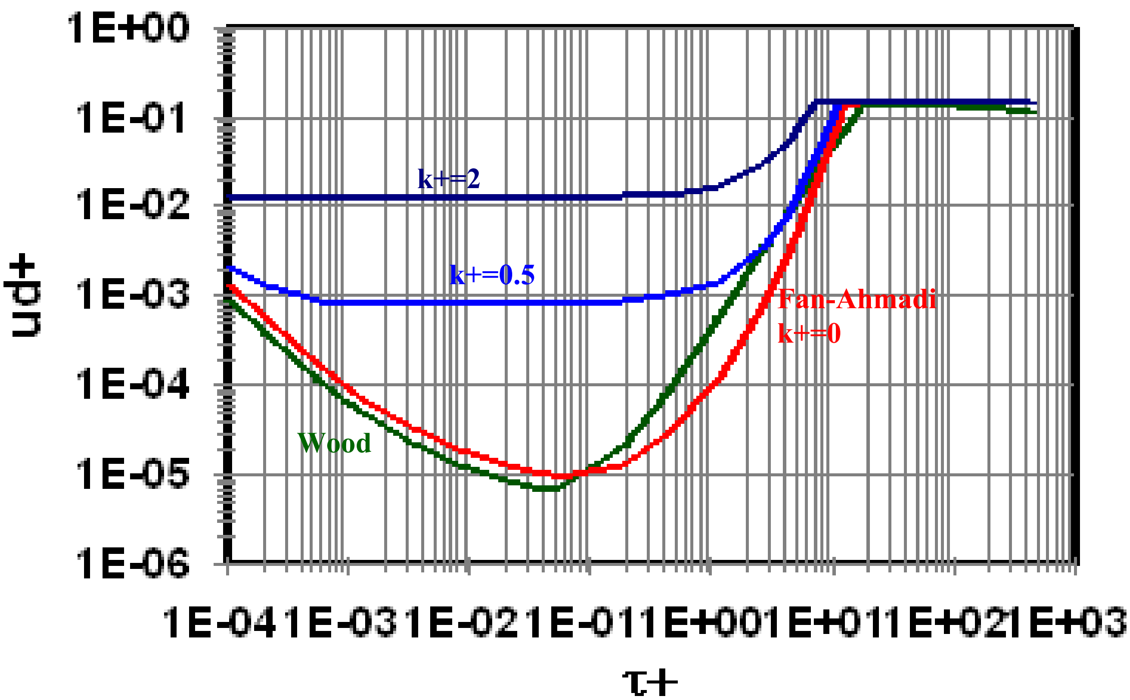

Figure 3. Variation of the nondimensional deposition velocity with nondimensional relaxation time for smooth and rough surfaces as predicted by Fan-Ahmadi formula.

Gravitational Deposition

Particles deposit on a horizontal surface due to the action of gravity. The gravitational deposition velocity is roughly equal to the terminal velocity. i.e.,

$$u_D \approx u^t = \tau g \tag{96}$$

or

$$u_D^+ = \tau^+ g^+, \tag{97}$$

where

$$g^+ = \frac{\nu g}{u^{*3}}. \tag{98}$$

Figure 4 compares the gravitational sedimentation velocity as given by Equation (97) with turbulent deposition and thermophoretic deposition for particles of different sizes under different flow and thermal conditions. It is seen that the gravitational effect is quite important for particles larger than a few micrometers, while the thermophoresis effects are significant for smaller particles. In the absence of thermal forces, the gravitational effects is significant for particles larger than 0.2 μm.

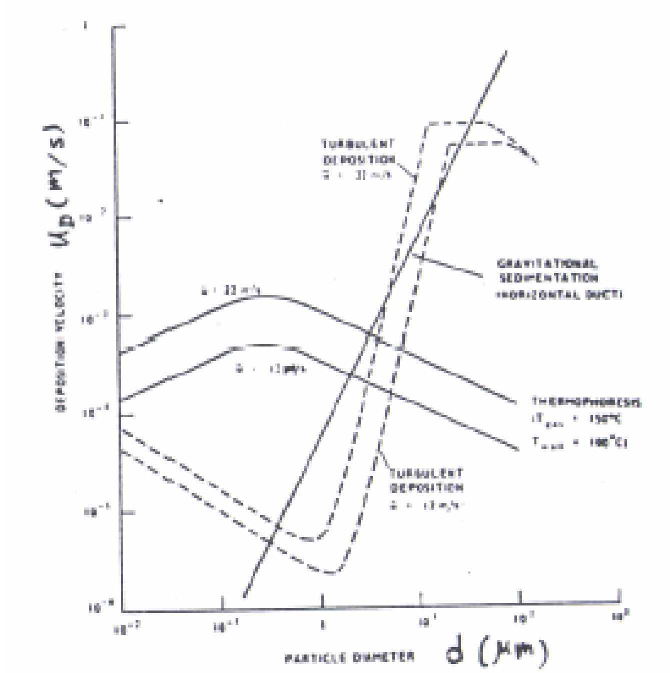

Figure 4. Comparison of deposition velocities due to turbulence, gravity and thermophoretic with particle diameter.

Figure 5 compares the gravitational deposition velocity with the turbulent deposition rate on a smooth wall as predicted by the models of Wood (1981) and Fan and Ahmadi (1993). Here a shear velocity of 0.3 m/s and particle-to-fluid density ratio of S=2000 are assumed. It seen that the gravitational sedimentation velocity is larger than turbulence deposition rate for particle relaxation times greater than 0.005 in wall units. For smaller particles, the Brownian diffusion dominates and the effect of gravity is negligible.

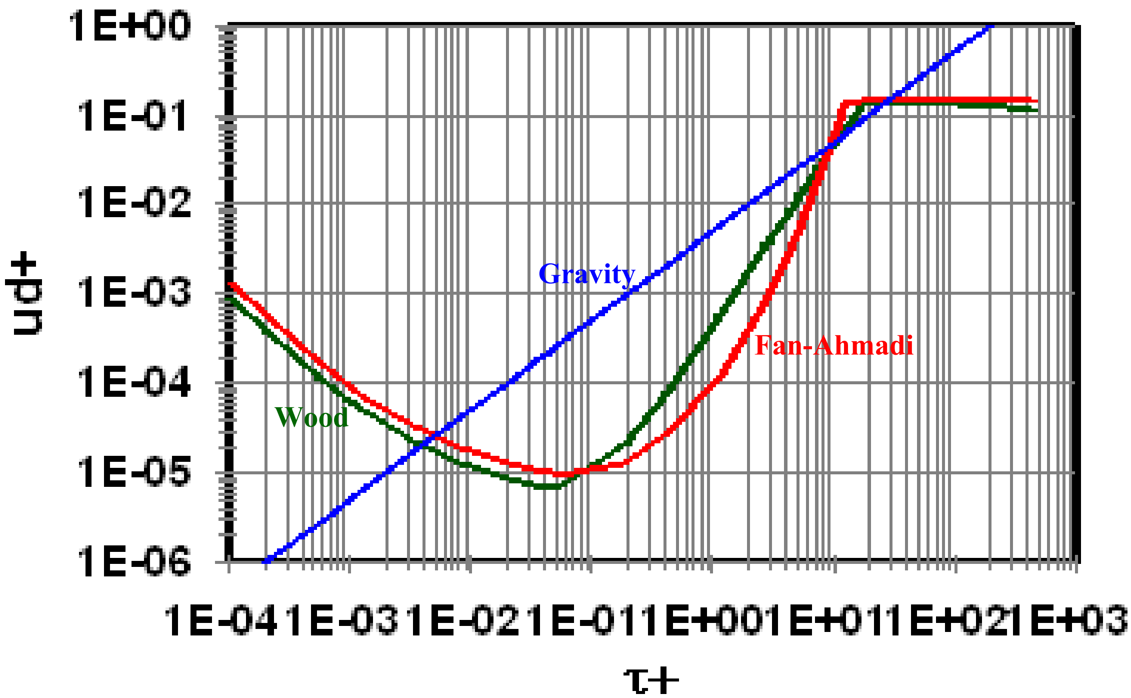

Figure 5. Variation of the nondimensional deposition velocity with nondimensional relaxation time for smooth and rough surfaces as predicted by Fan-Ahmadi formula.

Gaseous Mass Transfer in Turbulent Flows

Turbulence diffusion, usually, dominates the gaseous mass transfer processes. Davies obtained

$$u_D^+ = \frac{0.2}{s_c + \ln\left(\dfrac{s_c^{-1} + 5.04}{s_c^{-1} + 0.04}\right)}, \tag{99}$$

where s_c is the Schmidt number for the species considered. For $s_c >> 25$, Equation (99) reduces to

$$u_D^+ = \frac{0.2}{s_c + \ln(1 + 5.04 s_c)}. \tag{100}$$

For rough surfaces,

$$u_D^+ = \frac{0.2}{(1 - 0.09k^t)s_c + \ln(1 + 5.04 s_c)} \quad \text{for} \quad 0.45k^+ < 5 \tag{101}$$

and

$$u_D^+ = \frac{0.2}{\ln[(11.1 s_c^{-1} + 56)/k^t]} \quad \text{for} \quad 5 < 0.45k^+ << 30. \tag{102}$$

Note that when the wall is not absorbing and the concentration on the wall is C_w, then the mass flux is

$$J = U_D(C_0 - C_w) = U_D C_0 (1 - \frac{C_w}{C_0}), \tag{103}$$

where C_0 is the bulk concentration.