ME 637 - Particle -II

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Turbulence Deposition

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Turbulence Deposition

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Outline

- Semi-Empirical Models
- Free Flight Models
- Flow Structure Models
- Sublayer/Burst Models
- Deposition on Rough Walls

Turbulence Deposition Velocity

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Papavergos and Hedely (1984)

Vertical Channel

$\tau^+ < 0.2$ → $u_D^+ = 0.065 S_c^{-2/3}$

$0.2 < \tau^+ < 20$ → $u_D^+ = 3.5 \times 10^{-4} \tau^{+2}$

$\tau^+ > 20$ → $u_D^+ = 0.18$

Turbulence Deposition Velocity in a Vertical Duct

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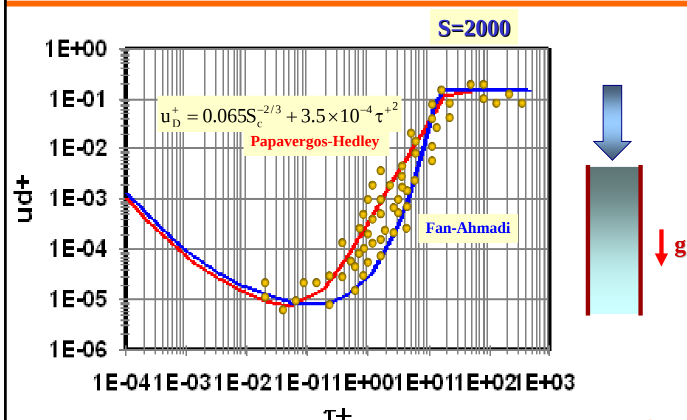

Turbulence Deposition Velocity

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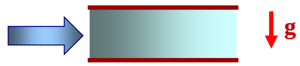
Papavergos and Hedely (1984)

Horizontal Duct



Floor

$$0.2 < \tau^+ < 20 \quad \Rightarrow \quad u_D^+ = 2\times10^{-3}\,\tau^{+2}$$

Ceiling

$$0.2 < \tau^+ < 20 \quad \Rightarrow \quad u_D^+ = 4\times10^{-5}\,\tau^{+2}$$

Turbulence Deposition Velocity in a Horizontal Duct

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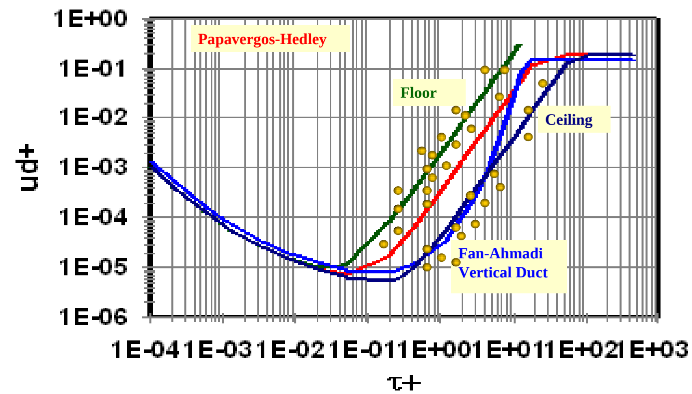

Turbulence Deposition Theories

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Mass Flux

$$J = (D + D^T)\frac{\partial C}{\partial y} \quad \Rightarrow \quad u_D^+ = (D^+ + D^{T+})\frac{\partial C^+}{\partial y^+}$$

Deposition Velocity

$$u_D^+ = \frac{u_D}{u^*} = \frac{1}{u^*}\left(\frac{J}{C_0}\right) \qquad C^+ = \frac{C}{C_0}$$

$$D^T = \nu^T \qquad D^{T+} = \nu^{T+}$$

Friedlander and Johnstone Model (Free Flight)

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Eddy Diffusivity

Friedlander and Johnstone (1959)

$$D^{T+} = \nu^{T+} = \begin{cases} \left(\dfrac{y^+}{14.5}\right)^3 & 0 \le y^+ \le 5 \\ \dfrac{y^+}{5} - 0.959 & 5 \le y^+ \le 30 \end{cases}$$

$$C\left(s^+ + \frac{d^+}{2}\right) = 0$$

Stopping Distance

$$s = U_f\,\tau$$

Free Flight Velocity

$$U_f = 0.9u^* = 0.9\bar{U}\sqrt{\frac{f}{2}}$$

$$f = C_f = \frac{\tau_0}{0.5\rho\bar{U}^2}$$

Friedlander and Johnstone Model

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Deposition Velocity

$s^+ = 0.9\tau^+$

$$u_d^+ = \left(\frac{1}{\sqrt{f/2}} + \frac{1525}{s^{+2}} - 50.6\right)^{-1} \quad s^+ \le 5$$

$$u_d^+ = \left[\frac{1}{\sqrt{f/2}} - 13.75 + 5\ln\left(\frac{5.04}{0.5s^+ - 0.959}\right)\right]^{-1} \quad 5 \le s^+ \le 30$$

$$u_d^+ = \sqrt{f/2} \quad s^+ \ge 30$$

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Turbulence Deposition Theories

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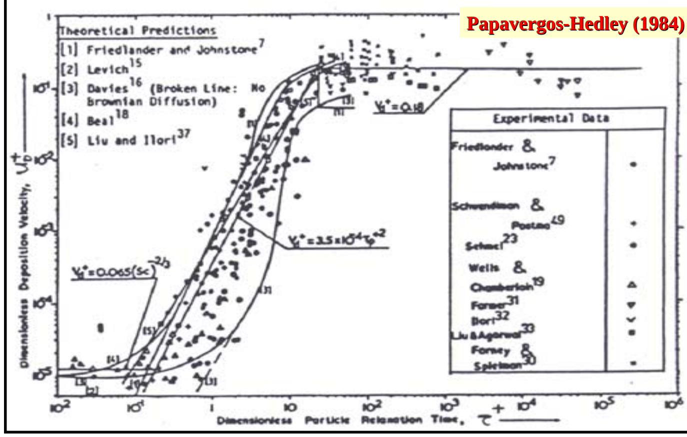
Papavergos-Hedley (1984)



Free Flight Deposition Models

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Levich (1962)

$\nu^T \sim y^{+4}$

$u_D^+ = 0.13337 S_c^{-3/4}$

Davies

$u_D^+ = 0.057 s_c^{-2/3}$

Sehmel

$$D^{T+} = 0.011(y^+)^{1.1}(\tau^+)^{1.1} \quad y^+ < 20$$

$$D^{T+} = 0.04y^+ \quad y^+ > 20$$

$U_f^+ = 1.49(\tau^+)^{-0.49}$

Liu and Ilory

$$D^{T+} = \nu^{T+} + \left(\frac{y^+}{y^+ + 10}\right)^2 \tau^+$$

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Comparisons of Turbulence Deposition Model

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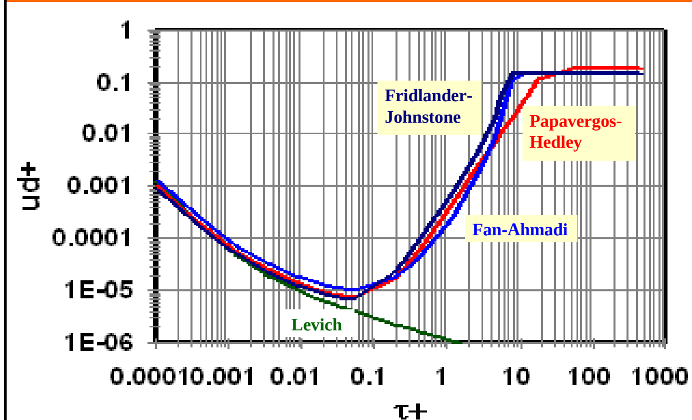

Limitations of Free-Flight Models

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- **Use of the concept of 'stopping distance' as a sink boundary condition for particle**
- **Assumptions for free-flight velocity**
- **Equality of particle mass diffusivity to the turbulence eddy diffusion.**
- **Ignoring the effects of density ratio, Reynolds number, and scales of turbulence.**
- **Ignoring the effects of lift force.**
- **Ignoring the effects of coherent eddies and bursting phenomena.**

Structure of Turbulence Near a Wall

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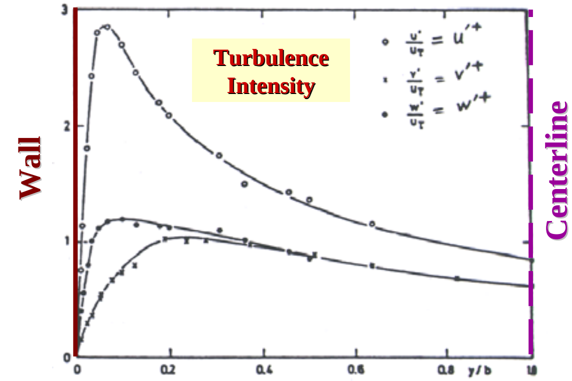

Streaky Wall Flow

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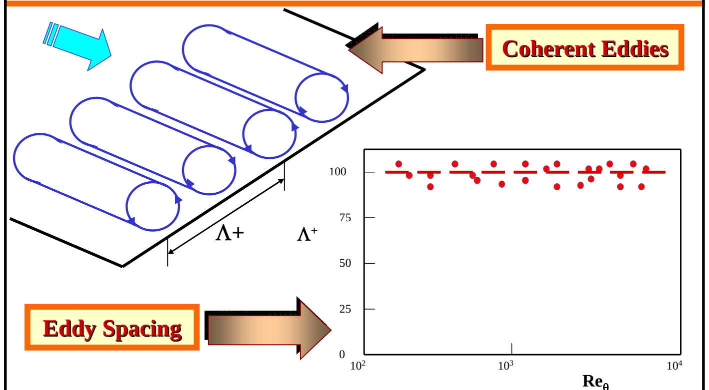

Bursting Phenomena

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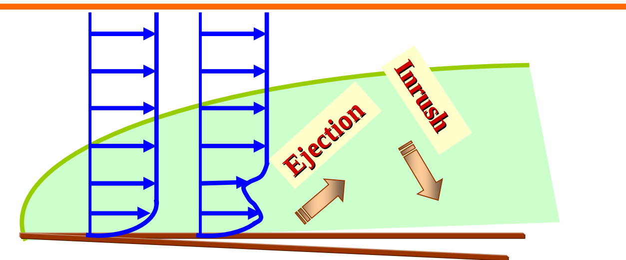

Time Between Bursts → $T_B^+ = 0.65 R_\theta^{0.73}$

$$\frac{V_0 T_B}{\delta} \approx 5$$

Bursts Duration $0.25 T_B$

Cleaver and Yates Model

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- **Suspended particles diffuse to a certain distance from the wall by turbulent diffusion before being entrained in a down-sweep.**
- **The flow in a down-sweep may be approximated as a two-dimensional stagnation-point flow in the sub-layer.**
- **Only Stokes drag is acting on the particles.**

$$u_D^+ = \frac{9}{400}\frac{\rho^f}{\rho^p}\tau^+ \exp\{0.48\tau^+\} + 0.084\,Sc^{-2/3}$$

Cleaver-Yates Model

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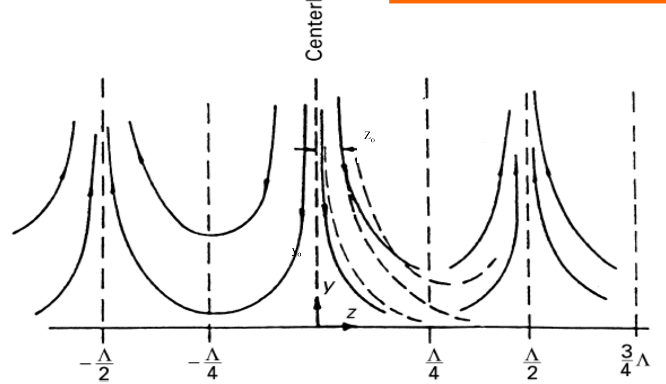
Chem. Eng. Sci. 30 (1975) 983.



Cleaver and Yates Model

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Flux → $J = C(y)v_0(y)A_c(y)$

Deposition Velocity → $u_D^+ = \frac{J}{Cu^*} = \frac{v_0^+ A_c}{2}$

Equation of Motion

A_c – capture area ratio $= \frac{z_0}{\Lambda}$

$$\tau^+\frac{dv^+}{dt^+} = v^{f+} - v^+$$

$$\tau^+\frac{dw^+}{dt^+} = w^{f+} - w^+$$

Cleaver and Yates Model

Clarkson University

Perturbation Solution

$$v^+ = v^{f+} - \tau^+\left(v^{f+}\frac{\partial v^{f+}}{\partial y^+} + w^{f+}\frac{\partial v^{f+}}{\partial z^+}\right)$$

$$w^+ = w^{f+} - \tau^+\left(v^{f+}\frac{\partial w^{f+}}{\partial y^+} + w^{f+}\frac{\partial w^{f+}}{\partial z^+}\right)$$

Flow Field

$w^f = \alpha z \varphi'(\eta)$ $v^f = -\sqrt{\alpha\nu}\,\varphi(\eta)$ $\eta = \sqrt{\frac{\alpha}{\nu}}\,y$

$$\varphi''' + \varphi\varphi'' - \varphi'^2 + 1 = 0$$

Cleaver and Yates Model

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$$\alpha = 0.067\frac{u^{*2}}{\nu}$$ ⟺ $y^+ = 10$, $v^{f+} = \frac{1}{2}$

Boundary condition → $y^+ = \frac{d^+}{2}$, $z^+ = 70$ → $A_c = \frac{z_0^+}{z^+}$

Limiting Trajectory →

$$\ln\frac{z^+}{z_0} = \int_{\sqrt{\frac{\alpha\nu}{u^{*2}}}y_0^+}^{\sqrt{\frac{\alpha\nu}{u^{*2}}}y^+} \left\{\frac{\varphi' - \tau^+(\frac{\alpha\nu}{u^{*2}})(\varphi'^2 - \varphi\varphi'')}{\varphi + \tau^+(\frac{d\nu}{u^{*2}})\varphi\varphi'}\right\}dy$$

Cleaver and Yates Model

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Deposition Velocity

$$u_D^+ = \frac{9}{400}\frac{\rho^f}{\rho^p}\tau^+\exp\{0.48\tau^+\}$$ $\tau^+ << 1$

$$u_D^+ = 0.45\exp\{-\frac{\tau}{0.9}\int_{d^+/2}^{\infty} 1 - \frac{f(y)}{0.9})dy\}$$ $\tau^+ >> 1$

To Account for Convection → $u_D^+ = 8.5A_c$

Deposition Velocity, Diffusion $u_D^+ = 0.084 S_c^{-2/3}$

Minimum Deposition Velocity → $\tau^+ S_c^{2/3} = 0.069\frac{\rho_p}{\rho_f}.$

Fichman et al. Model

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Flow Field $U^+ = y^+$

$$\left.\begin{array}{l} v^+ = B\phi = 0.625B^3y^{+2} \\ \phi = 0.625\eta^2, \quad \eta = By^+, \quad B = 0.271 \end{array}\right\} \text{for } y^+ \le 2$$

$$\left.\begin{array}{l} v^+ = c\phi = 0.24c - 0.71c^2y^+ \\ \phi = 0.71\eta - 0.24, \quad \eta = cy^+, \quad c = 0.174 \end{array}\right\} \text{for } 2 \le y^+ \le 7$$

$$\left.\begin{array}{l} U^+ = 0.3y^+ + 0.5 \\ v^+ = c\phi = 0.6c - c^2y^+, \ \phi = \eta - 0.6 \end{array}\right\} \text{for } 7 \le y^+ \le 30$$

Fichman et al. Model

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Deposition Velocity

$$u_D^+ = \frac{1}{2}A_c v_0^+$$

$$A_c = \frac{z_{lim}}{\Lambda/4} = \frac{z_{lim}^+}{\Lambda^+/4}$$

$$u_D^+ = \frac{2z_{lim}^+ v_0^+}{\Lambda^+}$$

Fichman et al. Model

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Equations of Motion

$$\tau^+ \frac{d^2x^+}{dt^{+2}} = U^+ - \frac{dx^+}{dt}$$

$$\tau^+ \frac{d^2y^+}{dt^{+2}} = v^+ - \frac{dy^+}{dt} + K\left(U^+ - \frac{dx^+}{dt}\right)$$

$$\tau^+ \frac{d^2z^+}{dt^{+2}} = w^+ - \frac{dz^+}{dt}$$

$$K = \tau^+ L^+$$

Lift → $L^+ = \frac{L\nu}{u^{*2}}$

Fichman et al. Model

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Lift → $$L \approx \frac{6.46\mu\left(\frac{d}{2}\right)^2}{\nu^{\frac{1}{2}} m_p}\left(\frac{dU}{dy}\right)^{\frac{1}{2}} = \frac{3.08\mu\dot{\gamma}^{\frac{1}{2}}}{\nu^{1/2} d\rho_p}$$

$$\dot{\gamma} = \frac{dU}{dy}$$

$$\frac{4z^+_{lim}}{\Lambda^+} = \frac{\frac{d^2}{4} + \left(d^+ + s^+\right)s^+}{4}$$

$$s^+ \le 2$$

$$s^+ = \frac{L^+\tau^{+2}\left(u^+_{po} - \dot{\gamma}^+ y^+_0\right) + \tau^+ v^+_{po}}{1 - \tau^{+2} L^+ \dot{\gamma}^+}$$

Numerical Solution → $s^+ \ge 2$

Cleaver and Yates Model

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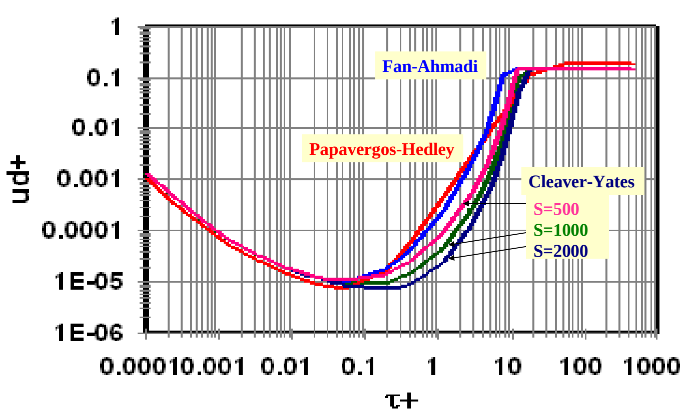

Fan-Ahmadi Model

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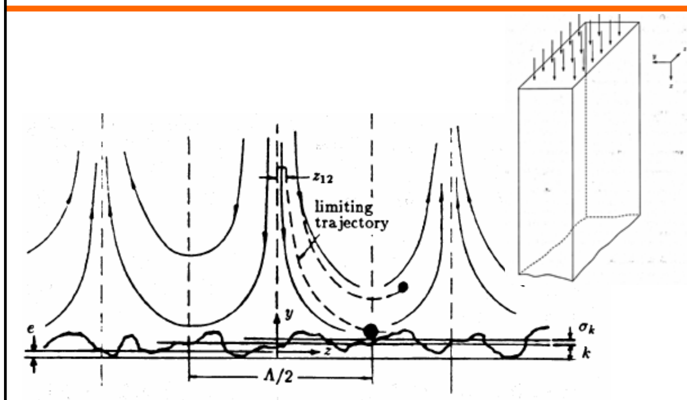

Fan-Ahmadi Model

Clarkson University

Equations of Motion

$$\tau^{+}\frac{du^{p+}}{dt^{+}}=u^{+}-u^{p+}+\tau^{+}g^{+},$$

$$\tau^{+}\frac{dv^{p+}}{dt^{+}}=v^{+}-v^{p+}+\tau^{+}L_1^{+}(u^{+}-u^{p+})+\tau^{+}L_2^{+}(w^{+}-w^{p+}),$$

$$\tau^{+}\frac{dw^{p+}}{dt^{+}}=w^{+}-w^{p+},$$

Fan-Ahmadi Model

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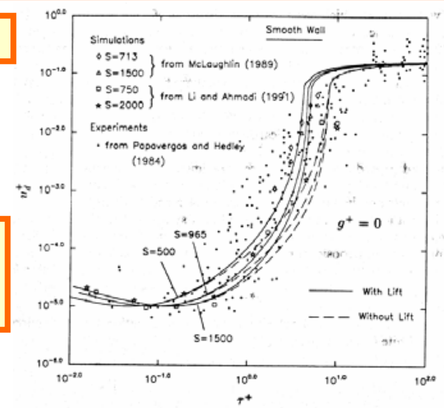
Smooth Wall

Comparison with Simulations



Fan-Ahmadi Model

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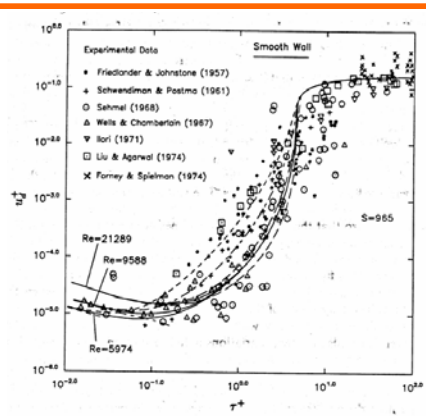
Smooth Wall

Comparison with Experiments

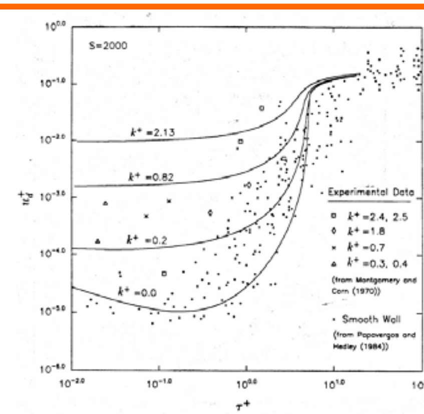


Fan-Ahmadi Model

Clarkson University

Rough Wall

Comparison with Experiments



Fan-Ahmadi Model

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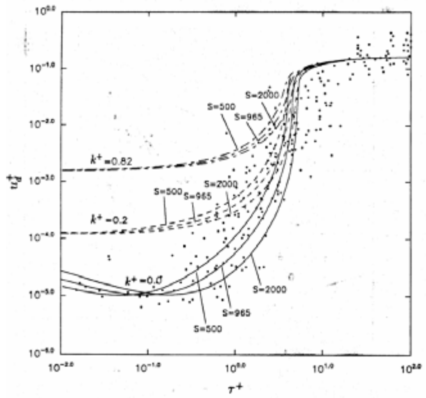
Rough Wall

Effects of Density Ratio



Deposition Velocity for Rough Walls

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Fan and Ahmadi

Empirical Model

$$u_d^+ = \begin{cases} \left\{ 0.084Sc^{-2/3} + \frac{1}{2}\left[\frac{\left(0.64k^+ + \frac{d^+}{2}\right)^2 + \frac{\tau_p^{+2} g^+ L_1^+}{0.01085(1+\tau_p^{+2}L_1^+)}}{3.42 + \frac{\tau_p^{+2} g^+ L_1^+}{0.01085(1+\tau_p^{+2}L_1^+)}} \right]^{1/\left(1+\tau_p^{+2}L_1^+\right)} \times \left[1 + 8e^{-(\tau_p^+ - 10)^2/32}\right] \frac{0.037}{1-\tau_p^{+2}L_1^+\left(1+\frac{g^+}{0.037}\right)} \right\} & \text{if } u_d^+ < 0.14 \\ 0.14 & \text{otherwise} \end{cases}$$

Fan-Ahmadi Model

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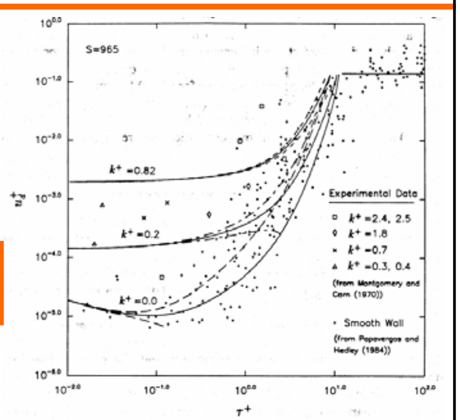
Rough Wall

Empirical Model Predictions



Concluding Remarks

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- **Semi-Empirical Models**
- **Free Flight Models**
- **Flow Structure Models**
- **Sublayer/Burst Models**
- **Deposition on Rough Walls**