Particle Transport, Deposition and Removal

INTRODUCTION TO AEROSOLS

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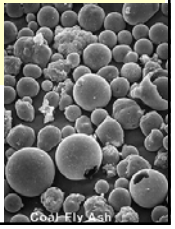
Outline

- Introduction to Aerosols
- Drag Forces
- Cunningham Corrections
- Lift Forces

Definition

- Aerosols are suspension of solid or liquid particles in a gas.
- Dust, smoke, mists, fog, haze, and smog are common aerosols.
- Aerosol particles are found in different shapes.



Aerosols in the Atmosphere

	Aerosols	Air
Number Density (Number/cm)	100-10^5	10^{19}
Mean Temperature (K)	240 – 310	240 – 310
Mean Free Path	Greater than 1 m	0.06 μm
Particle Radius	0.01 – 10 μm	2×10^{-4} μm
Particle Mass (g)	10^{-18} - 10^{-9}	4.6×10^{-23}
Particle Charge (Elementary Charge Units)	0 – 100	Weakly Ionized Single Charge

Dimensionless Groups

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Knudsen Number	$\mathrm{Kn} = \frac{2\lambda}{d}$
Mach Number	$M = \frac{\lvert \mathbf{v}^p - \mathbf{v}^f \rvert}{c^f}$
Schmidt Number	$\mathrm{Sc} = \frac{\nu}{D} = \frac{n^f \lambda d^2}{4}$
Brown Number	$\mathrm{Br} = \left(\frac{\overline{v^{p,2}}}{\overline{v^{f,2}}}\right)^{1/2} = \frac{\lvert \overline{v'^p} \rvert}{\lvert \overline{v'^f} \rvert}$
Reynolds Number	$\mathrm{Re} = \frac{\lvert \mathbf{v}^p - \mathbf{v}^f \rvert d}{\nu} = \frac{4M}{K_n}$

Relevant Parameters

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λ = Mean Free Path

d = Particle Diameter

v^p = Particle Velocity

v^f = Fluid (Air) Velocity

c = Speed of Sound

ν = Kinematic Viscosity

D = Diffusivity

v' = Thermal Velocity

n = Number Density

Mean Free Path

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$$\lambda = \frac{1}{\sqrt{2}\pi n d_m^2} = \frac{kT}{\sqrt{2}\pi d_m^2 P}$$

$$k = 1.38 \times 10^{-23}\ \mathrm{J/K}$$

Molecular Diameter

Air

$$\lambda(\mu m) = \frac{23.1T}{P}$$

Aerosols Characteristics

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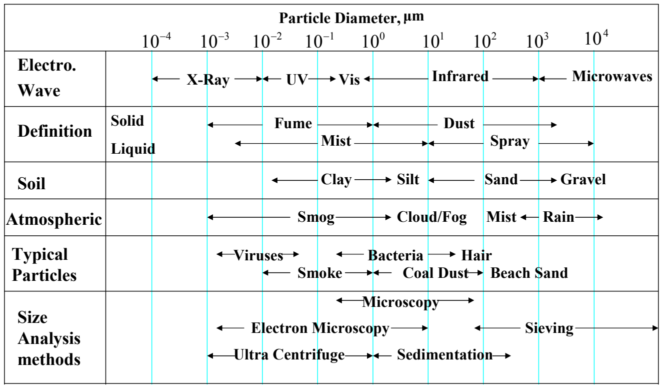
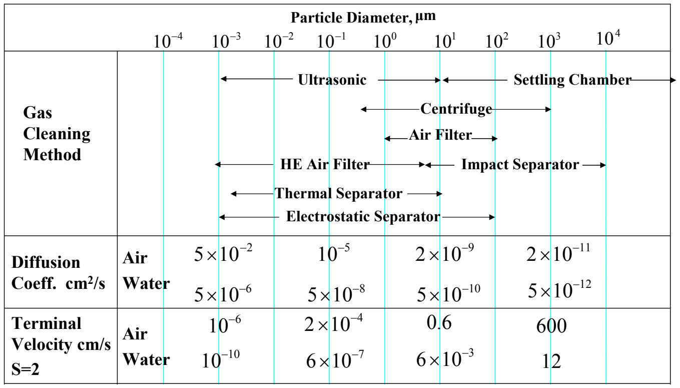


Aerosols Characteristics



		Particle Diameter, μm								
		10^{-4}	10^{-3}	10^{-2}	10^{-1}	10^{0}	10^{1}	10^{2}	10^{3}	10^{4}
Gas Cleaning Method		Ultrasonic Settling Chamber Centrifuge Air Filter HE Air Filter Impact Separator Thermal Separator Electrostatic Separator								
Diffusion Coeff. cm²/s	**Air**		5×10^{-2}		10^{-5}		2×10^{-9}		2×10^{-11}	
	Water		5×10^{-6}		5×10^{-8}		5×10^{-10}		5×10^{-12}	
Terminal Velocity cm/s S=2	**Air**		10^{-6}		2×10^{-4}		0.6		600	
	Water		10^{-10}		6×10^{-7}		6×10^{-3}		12	

Hydrodynamic Forces

Drag Forces

Stokes

$$F = 3\pi\mu U d$$

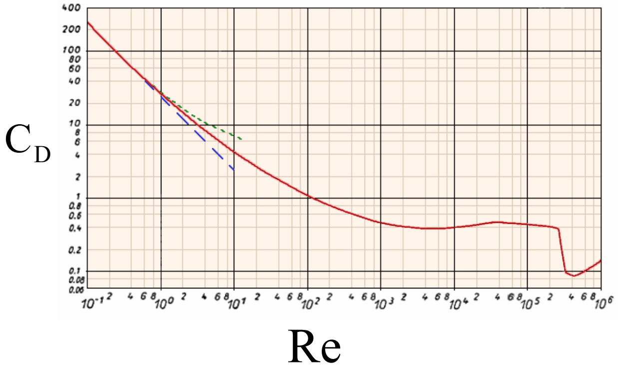
Drag Coefficient

$$C_D = \frac{F_D}{\frac{1}{2}\rho U^2 A} = \frac{24}{Re}$$

Reynolds Number

$$Re = \frac{\rho U d}{\mu}$$

Drag Force for a Sphere



Hydrodynamic Forces

Drag Forces

Oseen

$$C_D = \frac{24[1 + 3Re/16]}{Re}$$

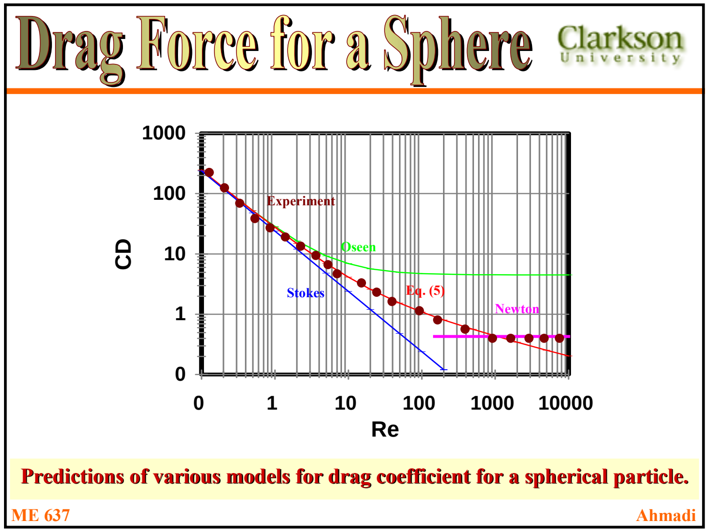
$1 < Re < 1000$

$$C_D = \frac{24[1 + 0.15 Re^{0.687}]}{Re}$$

Newton

$10^3 < Re < 2.5\times10^5$

$$C_D = 0.4$$



Drag Force for a Sphere

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Predictions of various models for drag coefficient for a spherical particle.

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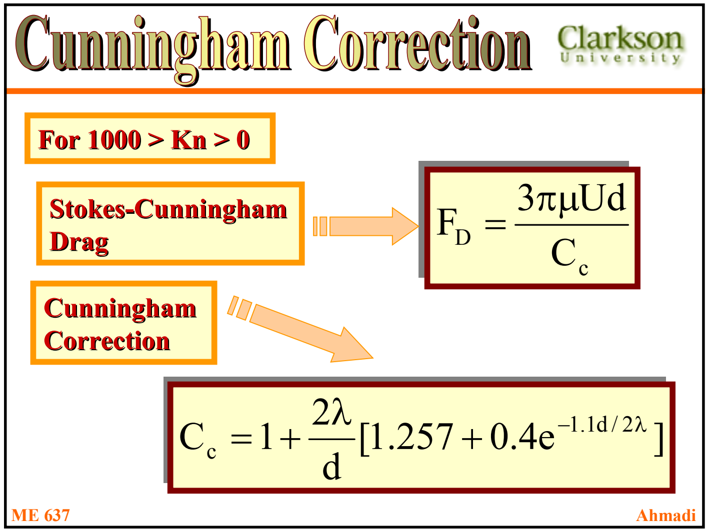

Cunningham Correction

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For 1000 > Kn > 0

Stokes-Cunningham Drag

$$F_D = \frac{3\pi\mu U d}{C_c}$$

Cunningham Correction

$$C_c = 1 + \frac{2\lambda}{d}[1.257 + 0.4e^{-1.1d/2\lambda}]$$

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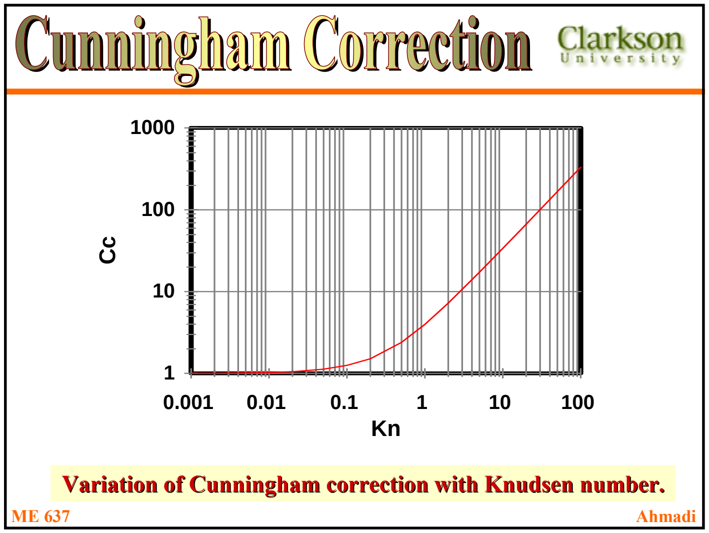

Cunningham Correction

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Variation of Cunningham correction with Knudsen number.

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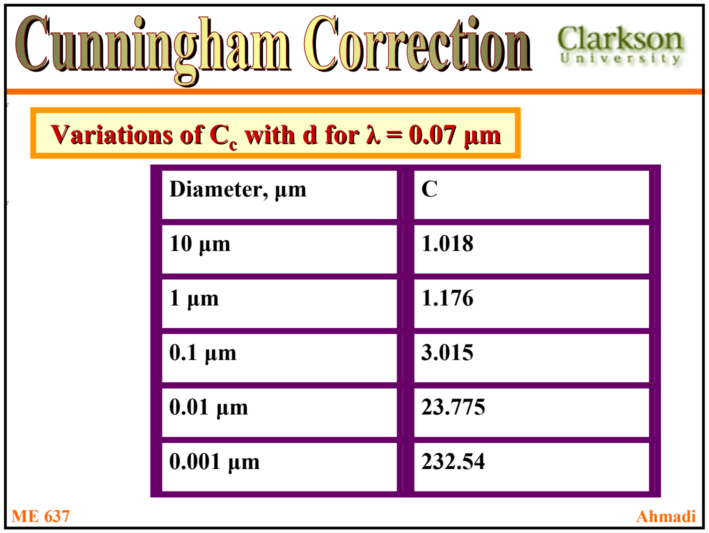

Cunningham Correction

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Variations of C_c with d for λ = 0.07 μm

Diameter, μm	C
10 μm	1.018
1 μm	1.176
0.1 μm	3.015
0.01 μm	23.775
0.001 μm	232.54

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Droplets

$$F_D = 3\pi\mu^f U d \frac{1+2\mu^f/3\mu^p}{1+\mu^f/\mu^p}$$

For Bubbles

$$F_D = 2\pi\mu^f U d$$

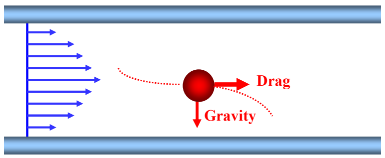
Non-Spherical Particles

$$F_D = 3\pi\mu U d_e K$$

$$d_e = \left(\frac{6}{\pi}\text{Volume}\right)^{1/3}$$

K=Correction Factor

Aerosols Particle Motion



Equation of Motion

$$m\frac{d\mathbf{u}^p}{dt} = \frac{3\pi\mu d}{C_c}(\mathbf{u}^f - \mathbf{u}^p) + mg$$

Aerosols Particle Motion

$$\tau\frac{d\mathbf{u}^p}{dt} = (\mathbf{u}^f - \mathbf{u}^p) + \tau\mathbf{g}$$

Relaxation Time

$$\tau = \frac{mC_c}{3\pi\mu d} = \frac{d^2\rho^p C_c}{18\mu} = \frac{Sd^2C_c}{18\nu}$$

$$S = \frac{\rho^p}{\rho^f}$$

$$\tau(s) \approx 3\times10^{-6} d^2(\mu m)$$

Terminal Velocity

$$\mathbf{u}^p = (\mathbf{u}^f + \tau\mathbf{g})(1 - e^{-t/\tau})$$

Terminal Velocity = Equilibrium Velocity after Large Time

$$u^t = \tau g = \frac{\rho^p d^2 g C_c}{18\mu}$$

$$u^t(\mu m / s) \approx 30\, d^2(\mu m)$$

Stopping Distance

Stopping Distance = Penetration distance for an initial velocity of u_o

$$\mathbf{u}^p = \mathbf{u}_o e^{-t/\tau}$$

$$\mathbf{x}^p = \mathbf{u}^p_o \tau(1 - e^{-t/\tau})$$

$$x^p = u^p_o \tau$$

$$x^p(\mu m) \approx 3\, d^2(\mu m)$$

Relaxation Time, Terminal Velocity and Stopping Distance

Diameter, μm	Terminal Velocity	τ sec	Stopping Distance u= 1 m/s	Stopping Distance u= 10 m/s
0.05	0.39 μm/s	4×10^{-8}	0.04 μm	0.0004 mm
0.1	0.93 μm/s	9.1×10^{-8}	0.092 μm	0.0009 mm
0.5	10.1 μm/s	1×10^{-6}	1.03 μm	0.0103 mm
1	35 μm/s	3.6×10^{-6}	3.6 μm	0.0357 mm
5	0.77 mm/s	7.9×10^{-5}	78.6 μm	0.786 mm
10	3.03 mm/s	3.1×10^{-4}	309 μm	3.09 mm
50	7.47 cm/s	7.6×10^{-3}	7.62 mm	76.2 mm

Particle Path

$$\mathbf{x}^p = \mathbf{x}^p_o + \mathbf{u}^p_o \tau(1 - e^{-t/\tau}) + (\mathbf{u}^f + \tau\mathbf{g})[t - \tau(1 - e^{-t/\tau})]$$

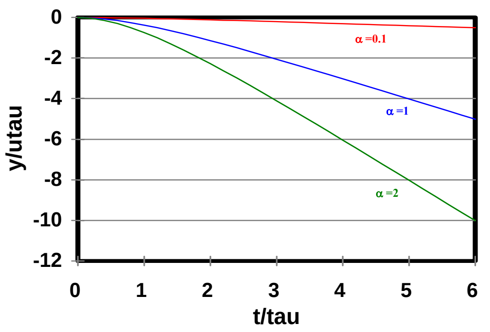
Components

$$x^p / u^f \tau = [t/\tau - (1 - e^{-t/\tau})]$$

$$y^p / u^f \tau = -\alpha[t/\tau - (1 - e^{-t/\tau})]$$

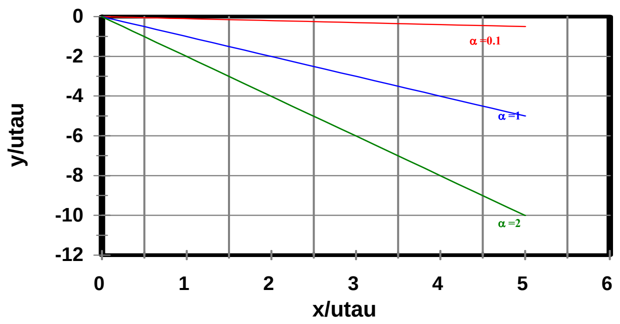
$$\alpha = \frac{\tau g}{u^f \tau}$$

Particle Path



Variations of the particle vertical position with time.

Particle Path



Sample particle trajectories.

Buoyancy Effects

$$(m+m^a)\frac{d\mathbf{u}^p}{dt} = \frac{3\pi\mu d}{C_c}(\mathbf{u}^f - \mathbf{u}^p) + (m - m^f)\mathbf{g}$$

Fluid Mass

$$m^f = \frac{\pi d^3 \rho^f}{6}$$

Apparent Mass

$$m^a = \frac{1}{2} m^f.$$

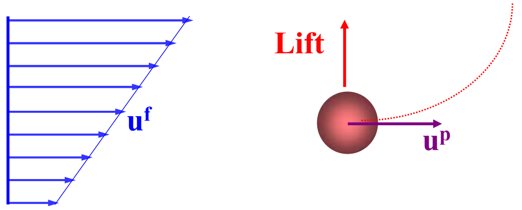
Buoyancy Effects

$$(1+\frac{1}{2S})\tau\frac{d\mathbf{u}^p}{dt} = (\mathbf{u}^f - \mathbf{u}^p) + \tau\mathbf{g}(1-\frac{1}{S})$$

Terminal Velocity

$$u^t = \tau g(1-\frac{1}{S}) = \frac{\rho^p d^2 g C_c}{18\mu}(1-\frac{\rho^f}{\rho^p})$$

Lift Force



Saffman (1965, 1968)

$$F_{L(Saff)} = 1.615 \rho \nu^{1/2} d^2 (u^f - u^p) \left| \frac{du^f}{dy} \right|^{1/2} \operatorname{sgn}\left(\frac{du^f}{dy}\right)$$

Saffman Lift Force Constraints

$$R_{es} = \frac{|u^f - u^p| d}{\nu} << 1$$

$$R_{e\Omega} = \frac{\Omega d^2}{\nu} << 1$$

$$R_{eG} = \frac{\dot{\gamma} d^2}{\nu} << 1$$

$$\varepsilon = \frac{R_{eG}^{1/2}}{R_{es}} >> 1$$

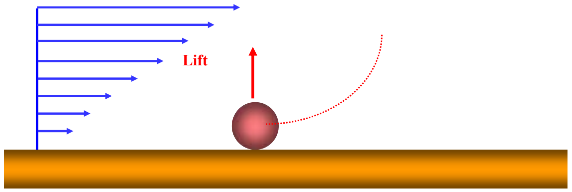
McLaughlin (1991)

Lift Force

McLaughlin (1991)

$$\frac{F_L}{F_{L(Saff)}} = \begin{cases} 1 - 0.287\varepsilon^{-2} & \text{for } \varepsilon >> 1 \\ -140\varepsilon^5 \ln(\varepsilon^{-2}) & \text{for } \varepsilon << 1 \end{cases}$$

Lift Force on a Particle Touching a Plane



Leighton and Acrivos (1985)

$$F_{L(L-A)} = 0.576 \rho d^4 \dot{\gamma}^2$$

Saffman

$$F_{L(Saff)} = 0.807 \rho \nu^{1/2} d^3 \dot{\gamma}^{3/2}$$

Lift Force in Turbulent Boundary Layer

$$\gamma = \frac{u^{*2}}{\nu}$$

$$F_L^+ = \frac{F_L}{\rho\nu^2}$$

$$d^+ = \frac{du^*}{\nu}$$

$$F^+_{L(L-A)} = 0.576 d^{+4}$$

$$F^+_{L(Saff)} = 0.807 d^{+3}$$

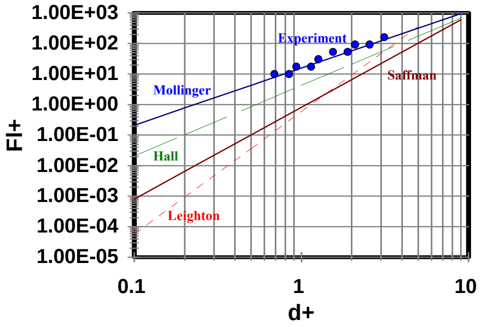
Hall (1988)

$$F^+_{L(Hall)} = 4.21 d^{+2.31}$$

Mollinger and Nieuwstadt (1996)

$$F^+_{L(MN)} = 15.57 d^{+1.87}$$

Lift Force in Turbulent Boundary Layer



Summary

- **Introduction to Aerosols**
- **Drag Forces**
- **Cunningham Corrections**
- **Lift Forces**

Thank you!

Questions?