Lift Force

Small particles in a shear field as shown in Figure 2 experience a lift force perpendicular to the direction of flow. The shear lift originates from the inertia effects in the viscous flow around the particle and is fundamentally different from aerodynamic lift force. The expression for the inertia shear lift was first obtained by Saffman (1965, 1968). That is

$$F_{L(Saff)} = 1.615 \rho \nu^{1/2} d^2 (u^f - u^p) \left| \frac{du^f}{dy} \right|^{1/2} \mathrm{sgn}\left(\frac{du^f}{dy}\right) \tag{1}$$

Here u^f is the fluid velocity at the location of mass center of the particle, u^p is the particle velocity, $\dot{\gamma} = \frac{du^f}{dt}$ is the shear rate, d is the particle diameter and ρ and ν are the fluid density and viscosity. Note that F_L is in the positive y-direction if $u^f > u^p$.

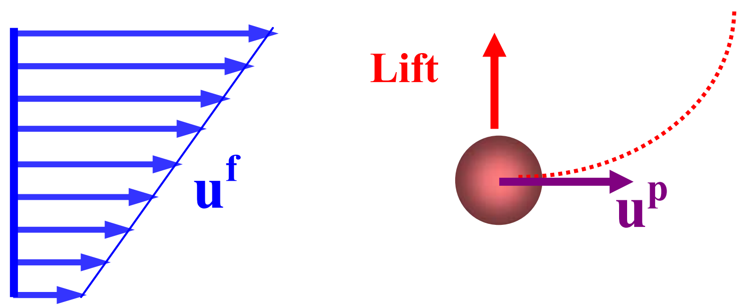

Figure 1. Schematics of a particle in a shear flow.

Equation (1) is subjected to the following constraints:

$$R_{es} = \frac{|u^f - u^p| d}{\nu} << 1 \qquad R_{e\Omega} = \frac{\Omega d^2}{\nu} << 1$$

$$R_{eG} = \frac{\dot{\gamma} d^2}{\nu} << 1 \qquad \varepsilon = \frac{R_{eG}^{1/2}}{R_{es}} >> 1$$

Here Ω is the rotational speed of the sphere. Dandy & Dwyer (1990) found that the Saffman lift force is approximately valid at larger R_{es} and small ε. McLaughlin (1991) showed that the lift force decreases as ε decreases. Based on these studies Mei (1992) suggested the following empirical fit to the results of Dandy and Dwyer and McLaughlin. For large ε and R_{es},

$$\frac{F_L}{F_{L(Saff)}} = \begin{cases} (1-0.3314\alpha^{1/2})\exp(-R_{es}/10) + 0.3314\alpha^{1/2} & \text{for } R_{es} \le 40 \\ 0.0524(\alpha R_{es})^{1/2} & \text{for } R_{es} > 40 \end{cases} \tag{2}$$

where

$$\alpha = \frac{\dot{\gamma} d}{2|u^f - u^p|} = \frac{R_{es}\varepsilon^2}{2} = \frac{R_{eG}}{2R_{es}} \tag{3}$$

For $0.1 \le \varepsilon \le 20$

$$\frac{F_L}{F_{L(Saff)}} = 0.3\{1 + \tanh[2.5\log_{10}(\varepsilon + 0.191)]\}\{0.667 + \tan[6(\varepsilon - 0.32)]\} \tag{4}$$

For large and small ε McLaughlin obtained the following expressions

$$\frac{F_L}{F_{L(Saff)}} = \begin{cases} 1 - 0.287\varepsilon^{-2} & \text{for } \varepsilon >> 1 \\ -140\varepsilon^5 \ln(\varepsilon^{-2}) & \text{for } \varepsilon << 1 \end{cases} \tag{5}$$

Note the change in sign of the lift force for small values of ε.

McLaughlin (1993) included the effects of presence of the wall in his analysis of the lift force. The results for particles in a shear field but not too close to the wall were given in tabulated forms. Cherukat and McLaughlin (1994) analyzed the lift force acting on spherical particles near a wall as shown in Figure 2. Accordingly

$$F_{L(C-L)} = \rho V^2 d^2 I_L / 4 \tag{6}$$

where

$$V = u^p - u^f = u^p - \dot{\gamma} l$$

and for non-rotating spheres,

$$\begin{aligned} I_L = {} & (1.7716 + 0.216K - 0.7292K^2 + 0.4854K^3) \\ & - (3.2397/K + 1.145 + 2.084K - 0.9059K^2)\wedge_G \\ & + (2.0069 + 1.0575 - 2.4007K^2 + 1.3174K^3)\wedge_G^2 \end{aligned} \tag{7}$$

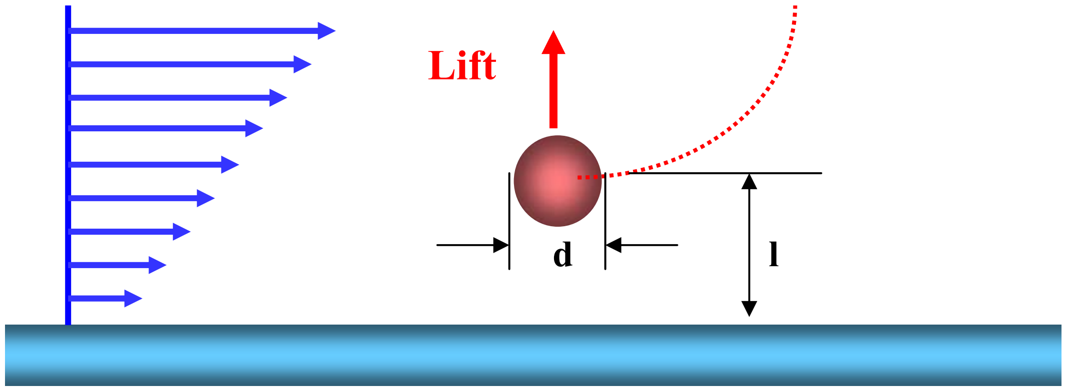

Figure 2. Schematics of a particle near a wall in a shear flow.

For rotating (freely) spheres,

$$\begin{aligned} I_L &= (1.7631s + 0.3561K - 1.1837K^2 + 0.845163K^2) \\ &- (3.24139/K + 2.6760 + 0.8248K - 0.4616K^2)\wedge_G \\ &+ (1.8081 + 0.879585K - 1.9009K^2 + 0.98149K^3)\wedge_G^2 \end{aligned} \tag{8}$$

Here

$$K = \frac{d}{2l}, \qquad \wedge_G = \frac{\dot{\gamma} d}{2v}. \tag{9}$$

Lift Force on a Particle Touching a Plane

Leighton and Acrivos (1985) obtain the expression for the lift on the spherical particles resting on a plane substrate as shown in Figure 4. They found

$$F_{L(L-A)} = 0.576\rho d^4\dot{\gamma}^2 \tag{10}$$

which is always point away from the wall. Note that the Saffman expression given by (1) may be restated as

$$F_{L(Saff)} = 0.807\rho\nu^{1/2} d^3\dot{\gamma}^{3/2} \tag{11}$$

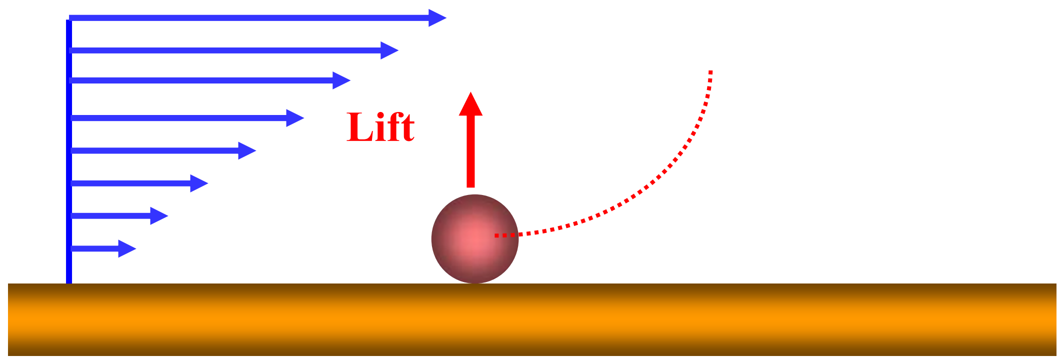

Figure 3. Schematics of a sphere resting on a wall in a shear flow.

Equation (6) with I_L given by (7) reduces to (10) for K = 1, Λ_G = -1.

For small particles in turbulent flows, using

$$u^+ = y^+ ,\ u^+ = \frac{u}{u^*} ,\ y^+ = \frac{yu^*}{\nu} ,\ \gamma = \frac{u^{*2}}{\nu} \tag{12}$$

where u^* is the shear velocity, equations (10) and (11) become

$$F^+_{L(L-A)} = 0.576 d^{+4} \tag{13}$$

$$F^+_{L(Saff)} = 0.807 d^{+3} \tag{14}$$

where

$$F^+_L = \frac{F_L}{\rho \nu^2} ,\qquad d^+ = \frac{du^*}{\nu} \tag{15}$$

Experimental studies of lift force were performed for generally larger particles in the range of 100 to several hundred μm. Hall (1988) found

$$F^+_{L(Hall)} = 4.21 d^{+2.31} \quad \text{for} \quad d^+ > 1.5 \tag{16}$$

Mollinger and Nieuwstadt (1996) found

$$F^+_{L(MN)} = 15.57 d^{+1.87} \ \text{for} \ \ 0.15 < d^+ < 1 \tag{17}$$

Figure 4 compares the model predictions with the experimental data of Hall. It is seen that the experimental data is generally much higher than the theoretical models.

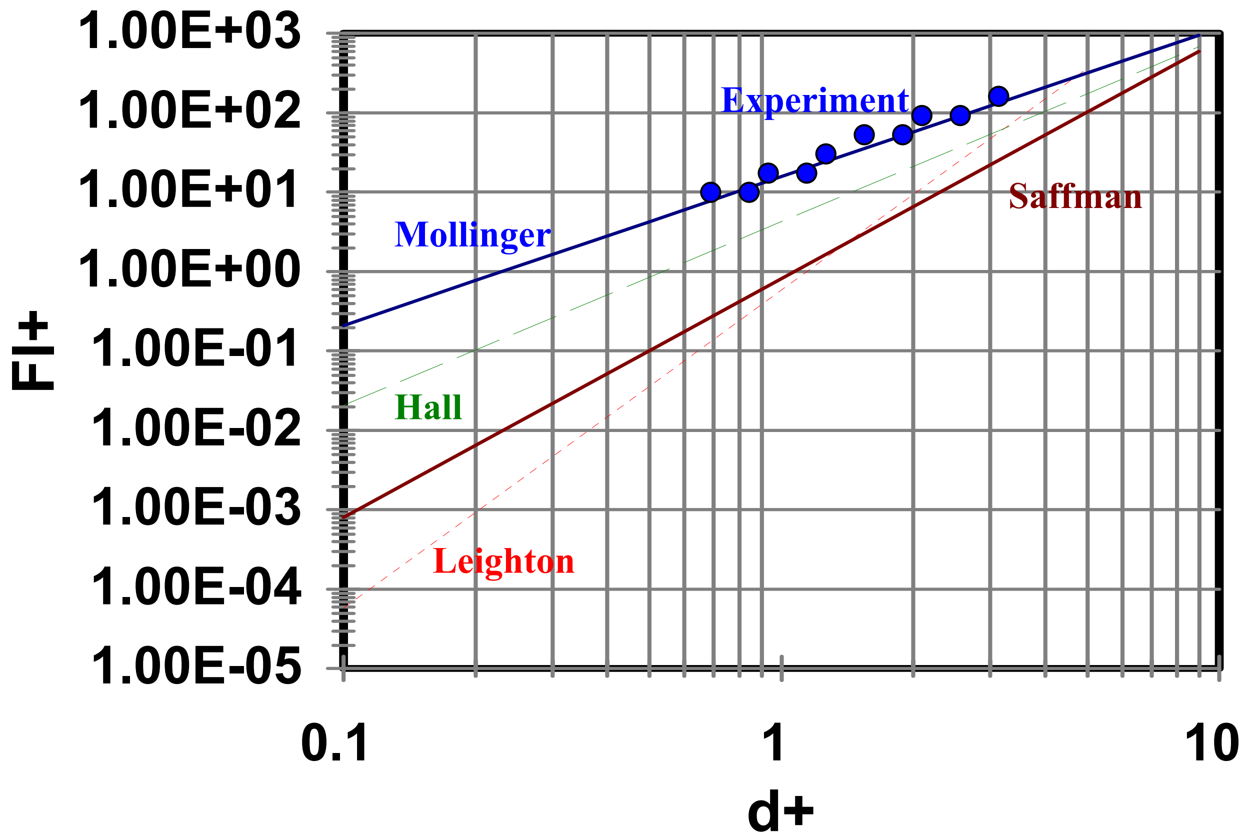

Figure 4. Comparison of model predictions with the experimental data.