

1. (40 points) Consider a steady convective-diffusion process with a flow velocity $\mathbf{V} = ay^2\mathbf{i}$ near an absorbing wall. The governing equation is given by

$$ay^2 \frac{\partial C}{\partial x} = D \frac{\partial^2 C}{\partial y^2}$$

where D is the diffusivity and a is a constant. The boundary conditions are:

$$C(0, y) = C_0, \quad C(x, \infty) = C_0 \quad \text{and} \quad C(x, 0) = 0.$$

- i. Use a similarity variable $\eta = \frac{y}{2(Dx/a)^{1/4}}$, reduce the governing equation and boundary conditions to the similarity form.
 - ii. Evaluate the concentration profile and the deposition velocity to the wall.
2. (35 points) Consider a 12 μm silicon particle that is attached to a silicon wafer in a turbulent air flow with a shear velocity of 2 m/s.
- i. Evaluate the drag, the Saffman lift and the hydrodynamic moment acting on the particle in wall units and in SI units.
 - ii. Evaluate the pull-off force as predicted by the JKR model.
 - iii. Find the contact radius at zero force and at the separation according to the JKR model.
 - iv. Is the particle going to be removed by the rolling mechanism? (Assume $u^+ = y^+$, and for silicon use $W_A = 0.0389 \text{ J/m}^2$, $E = 1.79 \times 10^{11} \text{ N/m}^2$, and Poisson ratio of 0.27. The kinematic viscosity of air is $\nu = 1.5 \times 10^{-5} \text{ m}^2/\text{s}$)
3. (25 points) Consider a cloud of 12 μm quartz particles with a concentration of 10^5 particles per cm^3 .

- i. Find the average absolute number of charge for the equilibrium Boltzmann distribution.
- ii. Determine the number of particles that will carry 5 positive charges. How many will carry no charges in this case?
- iii. Find the mean electrostatic precipitation velocity for a field of 400 Volt/cm for particles with the average absolute charge distribution.
- iv. Find the terminal velocity of these particles and compare with the electrostatic precipitation velocity.

(The density of air is 1.2 kg/m^3 , the density ratio of quartz particle to air is 2000, and charge of electron is $e = 1.59 \times 10^{-19} \text{ Coul.}$)