Clarkson University

Review for Final Exam

Goodarz Ahmadi

Department of Mechanical and Aeronautical Engineering
Clarkson University
Potsdam, NY 13699-5725

Outline

Clarkson University

- Hydrodynamic Forces and Moments
- Diffusion Mechanisms
- Particle Adhesion and Detachment
- Particle Charging

Hydrodynamic Forces

Clarkson University

Drag Forces

$$F_D = \frac{3\pi\mu U d}{C_c} C_D$$

$$Re = \frac{\rho U d}{\mu}$$

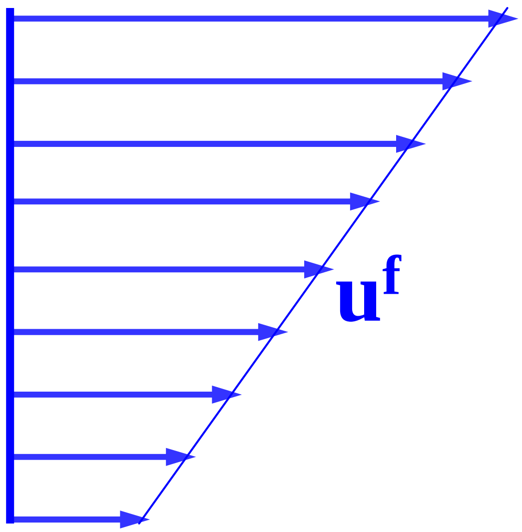
$$C_D = \frac{24[1 + 0.15\,Re^{0.687}]}{Re}$$

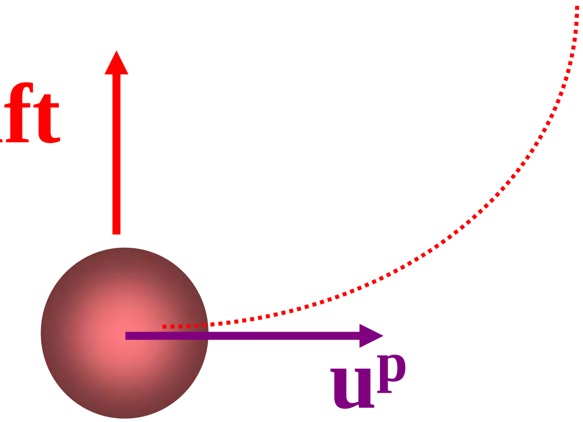
Cunningham Correction

$$C_c = 1 + \frac{2\lambda}{d}[1.257 + 0.4e^{-1.1d/2\lambda}]$$

Lift Force

Clarkson University





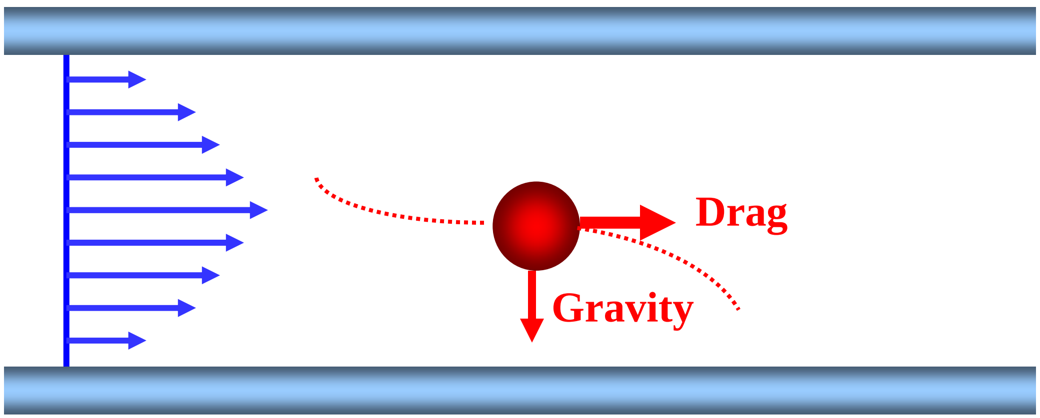
$$\text{sgn}(x) = \begin{cases} +1 & x > 0 \\ -1 & x < 0 \end{cases}$$

Saffman (1965, 1968)

$$F_{L(Saff)} = 1.615 \rho \nu^{1/2} d^2 (u^f - u^p) \left| \frac{du^f}{dy} \right|^{1/2} \text{sgn}\left(\frac{du^f}{dy}\right)$$

Aerosols Particle Motion

Clarkson University



Equation of Motion

$$m\frac{d\mathbf{u}^p}{dt} = \frac{3\pi\mu d}{C_c}(\mathbf{u}^f - \mathbf{u}^p) + m\mathbf{g}$$

Aerosols Particle Motion

Clarkson University

$$\tau \frac{d\mathbf{u}^p}{dt} = (\mathbf{u}^f - \mathbf{u}^p) + \tau \mathbf{g}$$

Relaxation Time

$$\tau = \frac{mC_c}{3\pi\mu d} = \frac{d^2 \rho^p C_c}{18\mu} = \frac{S d^2 C_c}{18\nu}$$

$$S = \frac{\rho^p}{\rho^f}$$

$$\tau(s) \approx 3 \times 10^{-6} d^2 (\mu m)$$

Viscous Sublayer

Clarkson University

Turbulent stress is negligible

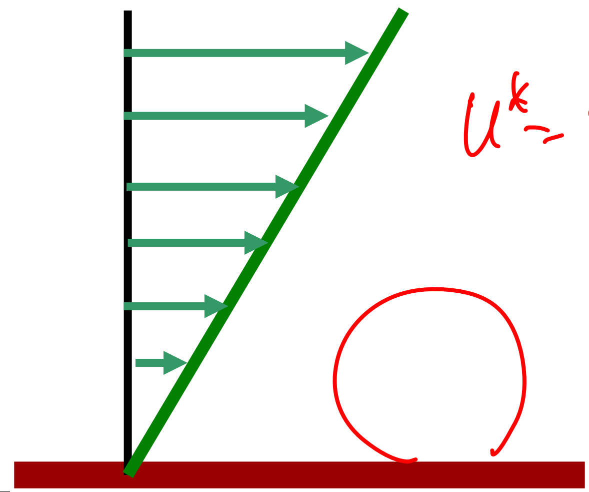
$$\tau_0 = \mu \frac{dU}{dy}$$

$$u^{*2} = \nu \frac{dU}{dy}$$

$$u = \frac{u^{*2} y}{\nu}$$

$$\frac{dU^+}{dy^+} = 1$$

$$u^+ = y^+$$



$u^* = \frac{1}{20} U_0$

u^* = shear velocity

$$0 < y^+ \le 5$$

ν, u^*

$F^+ = \frac{F}{\mu \nu}$

$$F^+_{L(Saff)} = 0.807 d^{+3}$$

$d^+ = \frac{d u^*}{\nu}$

Diffusion and Fick's Law

Clarkson University

Fick's Law

$$J = -D\frac{dc}{dx}$$

Diffusion Equation

$$\frac{\partial c}{\partial t} + \mathbf{v}\cdot\nabla c = D\nabla^2 c$$

Diffusivity

$$D = \frac{kTC_c}{3\pi\mu d}$$

Diffusion

Clarkson University

- Similarity Method
- Separation of Variable Method
- Integral Method

Particle Adhesion and Detachment

Clarkson University

- van der Waals Force
- JKR Adhesion Model
- DMT Adhesion Model
- Maugis-Pollock Model
- Particle Detachment Mechanisms
- Maximum Moment Resistance

JKR Model

Clarkson University

Johnson-Kandall-Roberts (1971)

$$a^3 = \frac{d}{2K}\left[P + \frac{3}{2}W_A \pi d + \sqrt{3\pi W_A dP + \left(\frac{3\pi W_A d}{2}\right)^2}\right]$$

$$K = \frac{4}{3}\left[\frac{1-\upsilon_1^2}{E_1} + \frac{1-\upsilon_2^2}{E_2}\right]^{-1}$$

Hertz Model

$$a^3 = \frac{dP}{2K}$$

DMT Model

Clarkson University

Derjaguin-Muller-Toporov (1975)

Pull-Off Force

$$F_{Po}^{DMT} = \pi W_A d$$

$$F_{Po}^{DMT} = \frac{4}{3} F_{Po}^{JKR}$$

Contact Radius at Zero Force

$$a_0 = \left(\frac{\pi W_A d^2}{2K} \right)^{\frac{1}{3}}$$

Contact Radius at Separation

$$a = 0$$

Maugis-Pollock Model

Clarkson University

$$P + \pi W_A d = \pi a^2 H$$

$$H = 3Y$$

$$a_0 \sim d^{\frac{2}{3}}$$ ↔ **Elastic**

$$a_0 \sim d^{\frac{1}{2}}$$ ↔ **Plastic**

JKR Model

Clarkson University

$$a^{*3} = 1 - P^* + \sqrt{1 - 2P^*}$$

$$P^* = -\frac{P}{\frac{3}{2}\pi W_A d}$$

$$a^* = \frac{a}{\left(\frac{3\pi W_A d^2}{4K}\right)^{\frac{1}{3}}}$$

$$M^{*JKR} = P^* a^* = P^* (1 - P^* + \sqrt{1 - 2P^*})^{1/3}$$

$$M_{max}^{*JKR} = 0.42$$

Aerosols Charging and Their Kinetics

Clarkson University

Coulomb Force

$$\mathbf{F}_E = q\mathbf{E}$$

$$q = ne$$

Particle Mobility

$$u = Z^p = \frac{qC_c}{3\pi\mu d}$$

Particle Charging

Clarkson University

Boltzmann Equilibrium Charge Distribution

$$f(n) = \frac{0.24}{\sqrt{d\pi}} \exp\{-\frac{0.05n^2}{d}\}$$

$$d > 0.02\mu m$$

$$\bar{n} = \approx 2.36\sqrt{d}, \quad d(\mu m)$$

Diffusion Charging

Clarkson University

$$n = \frac{dkT}{2e^2} \ln[1 + (\frac{2\pi}{m_i kT})^{1/2} n_{i\infty} d e^2 t]$$

Field Charging

$$n_\infty = [1 + \frac{2(\varepsilon_p - 1)}{\varepsilon_p + 2}] \frac{Ed^2}{4e} \text{ as } t \to \infty$$

ME 437/537 Final Exam Dec. 02

Problem 1

(40 points) Consider a steady convective-diffusion process with a flow velocity near an absorbing wall. The governing equation is given by

$$ay^2 \frac{\partial C}{\partial x} = D \frac{\partial^2 C}{\partial y^2}$$

where D is the diffusivity and a is a constant. The boundary conditions are:

$$C(x,0) = 0 \quad C(0,y) = C_0, \text{ and } \quad C(x,\infty) = C_0$$

i. Use a similarity variable, $\eta = \frac{y}{2(Dx/a)^{1/4}}$ reduce the governing equation and boundary conditions to the similarity form.

ii. Evaluate the concentration profile and the deposition velocity to the wall.

$$a y^2 \frac{\partial c}{\partial x} = D \frac{\partial^2 c}{\partial y^2} \qquad c = c(\eta)$$

Clarkson University

$$\frac{x}{y^2} \sim y^2 \qquad \left(\frac{y^4}{x}\right)^{1/4} \sim \eta \qquad \boxed{\eta = \frac{y}{2(Dx/a)^{1/4}}}$$

$$\frac{\partial c}{\partial y} = \frac{\partial c}{\partial \eta}\frac{\partial \eta}{\partial y} = \frac{\partial c}{\partial \eta}\left(\frac{1}{2(Dx/a)^{1/4}}\right)$$

$$\frac{\partial^2 c}{\partial y^2} = \frac{\partial^2 c}{\partial \eta^2}\frac{1}{4(Dx/a)^{1/2}}$$

$$\frac{\partial \eta}{\partial x} = \frac{y(-1/4)}{2(D/a)^{1/4}\, x^{5/4}} = -\frac{1}{4}\frac{\eta}{x}$$

$$\frac{\partial c}{\partial x} = \frac{\partial c}{\partial \eta}\frac{\partial \eta}{\partial x} = \frac{\partial c}{\partial \eta}\left(-\frac{1}{4}\frac{\eta}{x}\right)$$

$$\frac{\partial \eta}{\partial y} = \frac{1}{2\left(\frac{Dx}{a}\right)^{1/4}}$$

$$a y^2\left(-\frac{1}{4}\frac{\eta}{x}\right)\frac{\partial c}{\partial \eta} = D\frac{\partial^2 c}{\partial \eta^2}\frac{1}{4(Dx/a)^{1/2}}$$

$$-\frac{a y^2 \eta}{x D}\left(\frac{Dx}{a}\right)^{1/2}\frac{\partial c}{\partial \eta} = \frac{\partial^2 c}{\partial \eta^2} \rightarrow \boxed{-4\eta^3\frac{\partial c}{\partial \eta} = \frac{\partial^2 c}{\partial \eta^2}}$$

$$\frac{d^2c}{d\eta^2} + 4\eta^3 \frac{dc}{d\eta} = 0 \qquad C(0) = 0, \quad C(\infty) = C_0$$

Clarkson University

$$\frac{\frac{d^2c}{d\eta^2}}{\frac{dc}{d\eta}} = -4\eta^3 \rightarrow \ln \frac{dc}{d\eta} = -\eta^4 + \ln K$$

$$\frac{dc}{d\eta} = K e^{-\eta^4}$$

$$C = K\int_0^\eta e^{-\eta_1^4} d\eta_1 + A \nearrow 0 \qquad C(0) = 0$$

$$, \quad C_0 = K\int_0^\infty e^{-\eta_1^4} d\eta_1$$

$$C = C_0 \frac{\int_0^\eta e^{-\eta_1^4} d\eta_1}{\int_0^\infty e^{-\eta_1^4} d\eta_1}$$

$$J = D\frac{dc}{dy}\Big|_{y=0} = D\frac{dc}{d\eta}\Big|_{\eta=0} \frac{d\eta}{dy} = D\frac{C_0}{\int_0^\infty e^{-\eta_1^4} d\eta_1} \frac{1}{2(DX/a)^{1/4}}$$

$$u_d = \frac{J}{C_0} = \frac{D}{2\left(\frac{DX}{a}\right)^{1/4}} \frac{1}{\int_0^\infty e^{-\eta_1^4} d\eta_1}$$

Problem 2

Clarkson University

(35 points) Consider a 12 μm silicon particle that is attached to a silicon wafer in a turbulent air flow with a shear velocity of 2 m/s.

i. Evaluate the drag, the Saffman lift and the hydrodynamic moment acting on the particle in wall units and in SI units.

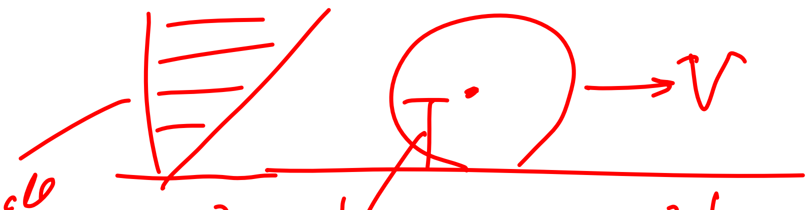
ii. Evaluate the pull-off force as predicted by the JKR model.

iii. Find the contact radius at zero force and at the separation according to the JKR model.

iv. Is the particle going to be removed by the rolling mechanism? (Assume $u^+ = y^+$, and for silicon use W_A= 0.0389 J/m^2, E = 1.79x10^{11} N/m^2, and Poisson ratio of 0.27. The kinematic viscosity of air is $\nu = 1.5 \times 10^{-5} m^2/s$)

$d = 12\,\mu m$, $u^* = 2\ m/s$, $d^+ = \frac{d u^*}{\nu} = \frac{12\times10^{-6}\times 2}{1.5\times10^{-5}}$

$d^+ = 1.6$



$$F_t = \frac{3\pi \mu d V f}{C_c}$$

$f = 1.7089$ (Eq. 22)

$C_D = 1$ Re - small

$u = \frac{u^{*2} y}{\nu}$, $\frac{d}{2}$, $V = \frac{u^{*2} d}{2\nu}$

$$F_t = \frac{3\pi(1.7)\mu d\, u^{*2} d}{2\nu C_c}$$

$$= \frac{3\pi(1.7)\rho d^2 u^{*2}}{2\, C_c} = \frac{2.9\pi \rho d^2 u^{*2}}{C_c} \quad (24)$$

$u^+ = y^+$, $V^+ = \frac{d^+}{2}$

$C_c = 1$ for $d = 12\,\mu m$

$$F_t^+ = \frac{F_t}{\mu\nu} = 2.9\pi \frac{d^{+2}}{C_c} = 9.11\, d^{+2} = 23.22$$

$$F_t = \mu\nu F_t^+ = 1.2(1.5\times10^{-5})^2\, 23.22 = 6.297\times10^{-9}\ N$$

$\mu = \rho\nu$, $\rho = 1.2$

Clarkson University

Lift

$F_L^+ = 0.807\, d^{+3} = 3.31$, $F_L = \mu \nu F_L^+ = 8.92 \times 10^{-10}\, N$

$$M = 1.07 \pi \rho \frac{u^{*2} d^3}{C_c} \quad (\text{Eq. } 30) = 3.36\, \rho u^{*2} d^3$$

$$M^+ = \frac{M}{\rho \nu^3 / u^*} = 3.36\, d^{+3} = 13.77$$

$$M = 2.79 \times 10^{-14}\, N \cdot m$$

ii) $F_{po}^{JKR} = \frac{3}{4} \pi W_A d = \frac{3}{4} \pi (0.0389)\, 1.2 \times 10^{-5} = 1.1 \times 10^{-6}\, N$

iii) $K = \frac{4}{3} \frac{E}{2(1-\nu^2)} = 1.287 \times 10^{11}\, N/m^2$

$$a_o = \left(\frac{3 \pi W_A d^2}{2K} \right)^{1/3} = 5.9 \times 10^{-8}\, m = 0.059\, \mu m$$

$$a = a_o / 4^{1/3} = 0.0372\, \mu m$$

iv) $F_D \frac{d}{2} + M_t + F_L a \geq M^{JKR}_{Max}$

$F_D \frac{d}{2} = 3.78\times10^{-14}$, $M_t = 2.79\times10^{-14}$, $F_L a = 3.32\times10^{-17}$, $M^{JKR}_{Max} = 4.1\times10^{-14}$

$$M^{Hydro} = 6.57\times10^{-14} > 4.1\times10^{-14} = M^{JKR}_{Max}$$

∴ Particle is Removed!

Problem 3

(25 points) Consider a cloud of 12 μm quartz particles with a concentration of 10^5 particles per cm^3.

i. Find the average absolute number of charge for the equilibrium Boltzmann distribution.

ii. Determine the number of particles that will carry 5 positive charges. How many will carry no charges in this case?

iii. Find the mean electrostatic precipitation velocity for a field of 400 Volt/cm for particles with the average absolute charge distribution.

iv. Find the terminal velocity of these particles and compare with the electrostatic precipitation velocity.

(The density of air is 1.2 kg/m^3, the density ratio of quartz particle to air is 2000, and charge of electron is $e = 1.59 \times 10^{-19}$ Coul.)

$d = 12\,\mu m, \quad C_0 = 10^5\ \#/cm^3$

Clarkson University

i) $\bar{n} = 2.36\sqrt{d} = 2.36\sqrt{12} = 8.17$

ii) $f(n) = \frac{0.24}{\sqrt{\pi d}} e^{-0.058 n^2/d} = 0.0391\, e^{-0.00483 n^2}$

$f(0) = 0.0391 \qquad N_0 = 3910$

$f(5) = 0.0346 \qquad N_0 = 3460$

iii) $u = \frac{E q C_c}{3 \pi \mu d}$

$$u = \frac{40000(8.17)(1.59\times10^{-19})}{3\pi(1.8\times10^{-5})\,1.2\times10^{-5}} = 2.55\times10^{-5}\ \frac{m}{s}$$

$$u = 25.5\ \mu m/s$$

iv) $$u_t = \tau g = \frac{S d^2}{18\nu} g = \frac{2000(12\times10^{-6})^2\cdot 9.81}{18(1.5\times10^{-5})}$$

$$u_t = 1.05\times10^{-2}\ m/s = 1.05\ cm/s$$