ME 637 **HW # 3**

1) (Problem 1.3, Tennekes and Lumley) Large eddies in turbulent flows have a length scale ℓ and a time scale $t(\ell) = \ell / u$. The smallest eddies have a length scale of η, a velocity scale of *L*, and time scale τ. Estimate the characteristic velocity *L*(r) and characteristic time t(r) of eddies of size r, where r is in the range of $\eta < r < \ell$. (Note that in this range *L*(r) and t(r) are determined by ε and r.) Show that your results agrees with the know results at $r = \eta$ and $r = \ell$. Find an express for the energy spectrum of turbulence, $E(\kappa) = \frac{v^2(\kappa)}{\kappa}$.

2) (Problem 3.1, Tennekes and Lumley) Estimate the characteristic velocity of eddies whose size is equal to the Taylor microscale λ. (See problem 1) Show that eddies of this size dissipates little energy.

3) Derive the energy equation for the Burger model

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \frac{\partial^2 u}{\partial x^2}$$

Assume $u = U + u'$. Discuss the meaning of the terms in the energy equation.

4) Consider a turbulent flow between two parallel plates. Derive the expression for the velocity in the viscous sublayer and in the log region. Assume the two solution should match at $y^+ = 10$. Assuming that the log profile is valid up to the channel centerline, find the expression for the friction coefficient

$$C_f = \frac{\tau_o}{\frac{1}{2}\rho U_c^2} = 2\left(\frac{u^*}{U_c}\right)^2$$