**ME 637** **Midterm Exam**

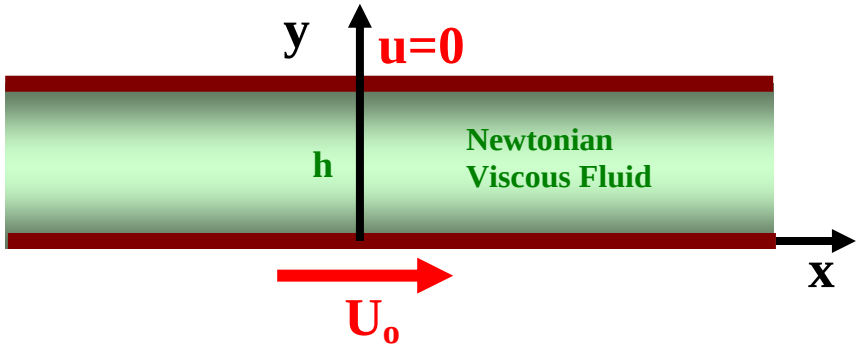
1. (20 Points) Consider the turbulent flow of an incompressible fluid. Estimate the order of magnitude of the following quantities in terms of u, λ, and Λ:

a) $\overline{u_i' \frac{\partial u_k'}{\partial x_i} u_k'}$ b) $\overline{\frac{\partial^2 \omega_i'}{\partial x_m \partial x_k} \frac{\partial^2 \omega_i'}{\partial x_m \partial x_k}}$ c) $U_j \overline{\frac{\partial \omega_i'}{\partial x_j}} U_i$ d) $\overline{\omega_i' \omega_j'} \cdot \overline{u_i' u_j'}$

2. (20 Points) Determine the contribution of eddies of size r to the correlations a) and b) in problem 1. For $r = \Lambda$, and $\eta$ evaluate these correlations, compare the contribution of large, and small eddies.

3. (30 Point) Consider a viscous Newtonian fluid between two long parallel walls. The lower wall is set suddenly in motion at time zero while the upper wall is stationary as shown in the figure. The fluid is initially at rest and there is no pressure gradient

   a. State the unsteady momentum and continuity equations for parallel the flow shown in it simplest form.
   b. State the boundary and initial conditions.
   c. Find the velocity field.
   d. Find the steady velocity profile in the duct.




4. (30 Points) Pollutant concentration and heat transport equations are given by

$$\frac{\partial c}{\partial t} + u_j \frac{\partial c}{\partial x_j} = \alpha \frac{\partial^2 c}{\partial x_j \partial x_j}, \qquad \frac{\partial T}{\partial t} + u_j \frac{\partial T}{\partial x_j} = \alpha \frac{\partial^2 T}{\partial x_j \partial x_j}$$

For turbulent flows, assume $c = C + c'$, $T = \overline{T} + T'$ and $u_i = U_i + u_i'$ where C, $\overline{T}$ and $U_i$ are mean values and c', T' and $u_i'$ are fluctuation quantities.

   a. Find the equation governing the average concentration C and average temperature $\overline{T}$.
   b. Find the transport equation for the concentration-Temperature correlations ($\overline{T'c'}$).
   c. Identify the terms in the equation for $\overline{T'c'}$.