

Turbulence Energy Equation

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Outline

- ▶ Mean Flow Energy Budget
- ▶ Turbulence Energy Budget
- ▶ Simple Shear Flows

Reynolds Equation

$$\rho \left(\frac{\partial U_i}{\partial t} + U_j \frac{\partial U_i}{\partial x_j} \right) = - \frac{\partial P}{\partial x_i} + \mu \frac{\partial^2 U_i}{\partial x_j \partial x_j} - \rho \frac{\partial \overline{u'_i u'_j}}{\partial x_j}$$

$$\frac{\partial U_i}{\partial x_i} = 0$$

$$\underbrace{\left[\frac{\partial}{\partial t} + U_j \frac{\partial}{\partial x_j} \right] \frac{U_i U_i}{2}}_{\text{convection}} = \underbrace{- \frac{\partial}{\partial x_i} \left[U_i \frac{P}{\rho} + U_j \overline{u'_i u'_j} \right]}_{\text{diffusion by turbulence}}$$

$$+ \underbrace{\nu \frac{\partial^2}{\partial x_j \partial x_j} \frac{U_i U_i}{2}}_{\text{viscous diffusion}} - \underbrace{\nu \frac{\partial U_i}{\partial x_j} \frac{\partial U_i}{\partial x_j}}_{\text{viscous dissipation}} + \underbrace{\overline{u'_i u'_j} \frac{\partial U_i}{\partial x_j}}_{\text{fluctuation energy production}}$$

$\frac{u^3/\Lambda}{\nu^2/\Lambda^2} \qquad \frac{u^3/\Lambda}{\nu^2/\Lambda^2} \qquad \frac{u^3/\Lambda}{\nu^2/\Lambda^2}$

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Navier-Stokes Eq. – Reynolds Equation ⇒

Fluctuation Velocity Equation

$$\frac{\partial u'_i}{\partial t} + U_j \frac{\partial u'_i}{\partial x_j} + u'_j \frac{\partial U_i}{\partial x_j} + u'_j \frac{\partial u'_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial p'}{\partial x_j} + \nu \frac{\partial^2 u'_i}{\partial x_j \partial x_j} + \overline{\frac{\partial u'_i u'_j}{\partial x_j}}$$

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$$\underbrace{\frac{\partial k}{\partial t} + U_j \frac{\partial k}{\partial x_j}}_{\text{Convection}} = \underbrace{-\overline{u'_i u'_j} \frac{\partial U_i}{\partial x_j}}_{\text{Production}} - \underbrace{\frac{\partial}{\partial x_j} \left[\frac{u'_j p'}{\rho} + \frac{u'_i u'_i u'_j}{2} \right]}_{\text{turbulent Diffusion}} + \underbrace{\nu \frac{\partial^2 k}{\partial x_j \partial x_j}}_{\text{Viscous Dissipation}} - \underbrace{\nu \overline{\frac{\partial u'_i}{\partial x_j} \frac{\partial u'_i}{\partial x_j}}}_{\text{Dissipation}}$$

$\frac{u^3/\Lambda}{\nu^2/\Lambda^2} \quad \frac{u^3/\Lambda}{\nu^2/\eta^2}$

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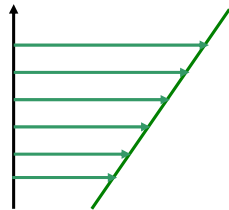
Energy Equation in a Pure Shear Flow Clarkson University

Turbulence Kinetic Energy Equation

$$\frac{D \overline{u'_i u'_i}}{Dt} = -\frac{\partial}{\partial x_j} \left[u'_j \left(\frac{p'}{\rho} + \frac{u'_i u'_i}{2} \right) \right] - \overline{u'_i u'_j} \frac{\partial U_i}{\partial x_j} - \epsilon + \frac{p'}{\rho} \frac{\partial u'_j}{\partial x_j}$$

$$\mathbf{U} = (U_1(y), 0, 0)$$

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial z} = \mathbf{U} \cdot \nabla = 0$$



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$$0 = -\frac{\partial}{\partial y} \left[u'_2 \left(\frac{p'}{\rho} + \frac{u'_1 u'_1}{2} \right) \right] - \overline{u'_1 u'_2} \frac{\partial U_1}{\partial y} - \epsilon + \frac{p'}{\rho} \frac{\partial u'_j}{\partial x_j}$$

$$\frac{1}{2} \overline{u_1'^2} \Rightarrow 0 = -\frac{\partial}{\partial y} \left(\frac{u'_2 u'_1 u'_1}{2} \right) - \overline{u'_1 u'_2} \frac{\partial U_1}{\partial y} - \frac{1}{3} \epsilon + \frac{p'}{\rho} \frac{\partial u'_1}{\partial x}$$

$$\frac{1}{2} \overline{u_2'^2} \Rightarrow 0 = -\frac{\partial}{\partial y} \left[u'_2 \left(\frac{p'}{\rho} + \frac{u'_2 u'_2}{2} \right) \right] - \frac{1}{3} \epsilon + \frac{p'}{\rho} \frac{\partial u'_2}{\partial y}$$

$$\frac{1}{2} \overline{u_3'^2} \Rightarrow 0 = -\frac{\partial}{\partial y} \left[u'_2 \left(\frac{u'_3 u'_3}{2} \right) \right] - \frac{1}{3} \epsilon + \frac{p'}{\rho} \frac{\partial u'_3}{\partial z}$$

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Concluding Remarks

- ▶ Mean Flow Energy Budget
- ▶ Turbulence Energy Budget
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Thank you!

Questions?