Inertia Impactions

Near the stagnation point of an in-viscid flow the fluid velocity field is given by

$$U = -bV\left(\frac{x}{a} + 1\right), \tag{1}$$

where b is a non-dimensional constant. For a particle under Stokes drag moving on the stagnation streamline, the equation of motion is given by

$$m\ddot{x} = -3\pi\mu d(\dot{x} - u). \tag{2}$$

Using (1) in (2) and restating the resulting equation in a non-dimensional form, it follows that

$$Stk\frac{d^2x^*}{dt^{*2}} + \frac{dx^*}{dt^*} + bx^* = 0, \tag{3}$$

where the Stokes number is defined as

$$Stk = \frac{\rho^p d^2 U}{18\mu a} = \frac{\tau^p U}{a}, \tag{4}$$

and

$$x^* = \frac{x}{a} + 1, \qquad t^* = t\frac{U}{a}. \tag{5}$$

For the linear constant coefficient equation given by (3), the solution is given as

$$x^* = Ae^{\lambda_1 t} + Be^{\lambda_2 t}, \tag{6}$$

where λ_1 and λ_2 are the solution to the characteristic equation given by

$$Stk\,\lambda^2 + \lambda + b = 0. \tag{7}$$

That is,

$$\lambda = -\frac{-1 \pm \sqrt{1 - 4bStk}}{2Stk} \tag{8}$$

It is observed that for

$$Stk < Stk_{crit} = \frac{1}{4b} \tag{9}$$

the roots are real and negative. This will lead to vanishing of $\frac{dx^*}{dt^*}$ at $x^* = 0$ and hence zero impaction efficiency. For $St > St_{crit}$, however, $\frac{dx^*}{dt^*}$ is positive at $x^* = 0$ leading to finite collection efficiency. For an inviscid flow around cylinders $b = 2$, and $Stk_{crit} = 1/8$. For spheres, $b = 3$ and $Stk_{crit} = 1/12$. Additional details were provided by Friedlander (2000).

Impaction of Non-Stokesian Particles

The equation of motion of a particle is given as

$$m\frac{d\mathbf{u}^p}{dt} = -\frac{C_D R_e}{24}(3\pi\mu d)(\mathbf{u}^p - \mathbf{u}^f), \tag{10}$$

where

$$R_e = \frac{d\,|\mathbf{u}^p - \mathbf{u}^f|}{\nu}, \tag{11}$$

and C_D is a drag coefficient. Equation (10) may be restated in non-dimensional form as

$$\frac{d\mathbf{u}^{p*}}{dt} = -\frac{C_D\,Re_d}{24Stk}|\mathbf{u}^{p*} - \mathbf{u}^{f*}|(\mathbf{u}^{p*} - \mathbf{u}^{f*}). \tag{12}$$

Numerical solutions for inviscid flows around cylinders and spheres were obtained by Brun et al. (1955) and Dorsch et al. (1955). The results depend on Stk and parameter P given as

$$P = \frac{Re_d^2}{Stk} = \frac{18(\rho^f)^2\,Ua}{\mu\rho^p}, \tag{13}$$

where the particle Reynolds number is defined as

$$Re_d = \frac{Ud}{\nu}. \tag{14}$$

<u>Variations of capture efficiency with Stokes number for different values of P were given by Frielander (2000).</u>

Deposition by Inertia Impaction

Impaction on Sphere in Cross Flow

In this section deposition on a sphere in cross flow as shown in Figure 1 is studied.

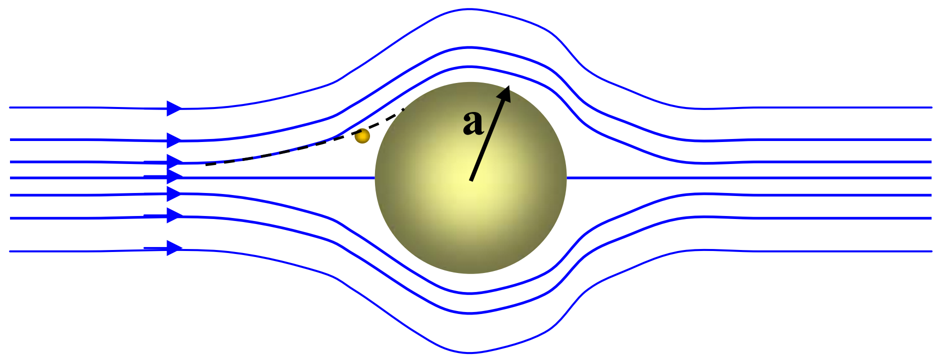

Figure 1. Particle deposition by impaction on a sphere in cross flows.

For particle impaction on a sphere in cross flows, assuming an inviscid flow model, Langmair and Bladgett (1948) found that the capture efficiency is given by

$$\eta = \frac{\mathrm{Stk}^2}{(\mathrm{Stk} + 0.25)^2} \tag{1}$$

where Stokes Number is defined as

$$\mathrm{St} = \frac{\rho^p d^2 U}{18 \mu a} = \frac{\tau^p U}{a}\,. \tag{2}$$

Here a is the radius of the sphere, U is the flow velocity, d is the particle diameter, and τ^p is the particle relaxation time. The variation of capture efficiency as given by Equation (1) is shown in Figure 2, and the result is compared with the experimental data of Walton and Woolcock (1960), and the prediction of a viscous flow model. It is seen that the inviscid flow model given by Equation (1) is in reasonable agreement with the data of Walton and Woolcock (1960). The viscous flow model underestimates the experimental data. The earlier data of Ranz and Wong (1952) suggest higher level of deposition that is predicted by the inviscid flow model.

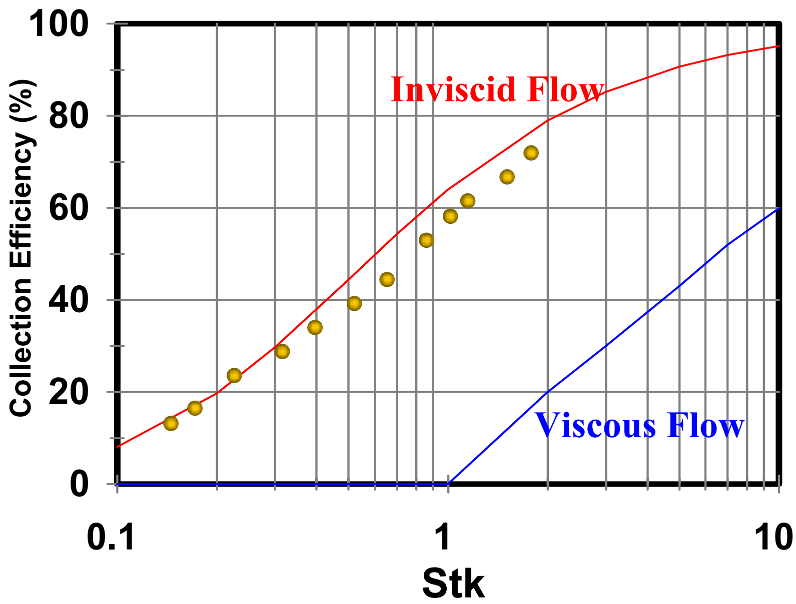

Figure 2. Variation of capture efficiency on a sphere with Stk number. Comparison with the experimental data of Walton and Woolcock (1960).

Impaction on Cylinder in Cross Flow

For particle impaction on a cylinder in cross flows, Langmair and Bladgett (1948) suggested

$$\eta = \begin{cases} \dfrac{St^2}{(St+0.6)^2} & St > 0.08 \\ 0 & St < 0.08 \end{cases} \tag{3}$$

where an inviscid flow model was used. Similarly, Davies (1952) used a viscous flow model and found

$$\eta = 0.16[\frac{d}{2a} + (0.25 + 0.2\frac{d}{a})St - 0.0263\frac{d}{2a}S_t^2] \tag{4}$$

Here a Reynolds number less than 1 is assumed.

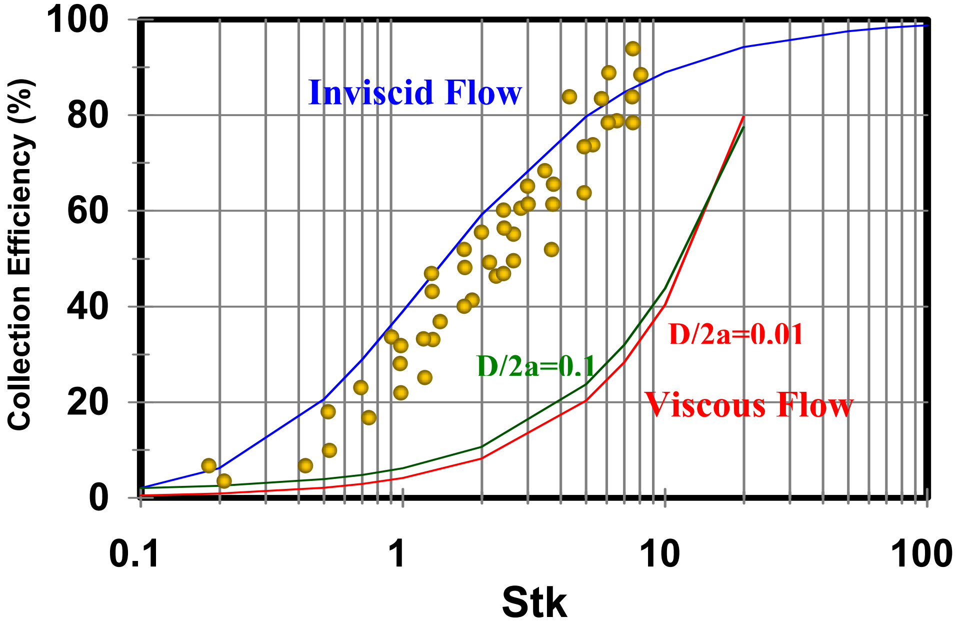

Figure 3. Variation of capture efficiency of a cylinder with Stk number. Comparison with the experimental data of Ranz and Wong (1952).

References

Ranz, W. and Wong, J. Ind. Eng.Chem. Vol. 44, p. 844 (1952).
Walton, W. and Woolcock, A., Aerodynamic Capture of particles, Pergamon Press, New York (1960).
Davies, C.N. Proc. Inst, Mech. Engineering, Vol. 1B, p. 185 (1952)