ME437/ME537 HW SET 2

1. For a particle of mass m in an air stream with a constant velocity $\mathbf{u}^{\mathbf{f}}$ and in gravitational field, evaluate the velocity and position vectors of the particle as a function of time. Also evaluate the terminal velocity of the particle when a constant velocity $\mathbf{u}^{\mathbf{f}}$ is present.
2. Show that the mean square response of particle position given by $\overline{x^2(t)} = \int_0^t \int_0^t R_{uu}(\tau_1 - \tau_2) d\tau_1 d\tau_2$ may be restated as $\overline{x^2(t)} = 2\int_0^t (t-\tau) R_{uu}(\tau) d\tau$, and the diffusivity is given by $D = \int_0^\infty R_{uu}(\tau) d\tau$.
3. Evaluate the concentration of uniform size spheres with a constant terminal velocity v^t. Assume that $c = c_o$ at $x = x_o$, and $c = c(x)$ with x being in the vertical direction. Note that the generalized mass diffusion equation is given as $\frac{\partial c}{\partial t} + (\mathbf{v} + v^t) \cdot \nabla c = D \nabla^2 c$
4. Consider the case of diffusion to a wall governed by $\frac{\partial c}{\partial t} = D \frac{\partial^2 c}{\partial y^2}$, $c(0,t) = 0$, $c(\infty, t) = c_o$ and $c(y,0) = c_o$. Use the integral method with an approximate expression for the profile given by $\frac{c}{c_o} = 2\frac{y}{\delta_c} - (\frac{y}{\delta_c})^2$ and evaluate the variation of diffusion boundary layer thickness δ_c with time.
5. Evaluate the variation of concentration with space and time in a region between two parallel plates with an initially uniform concentration. Note that $\frac{\partial c}{\partial t} = D \frac{\partial^2 c}{\partial y^2}$, $c(0,t) = 0$, $c(b,t) = 0$ and $c(y,0) = c_o$. [Hint: use the method of separation of variables.]
6. The diffusion equation in cylindrical coordinate is given as $\frac{\partial c}{\partial t} = \frac{D}{r} \frac{\partial}{\partial r}(r \frac{\partial c}{\partial r})$. Reduce the diffusion equation to a similarity form by assuming that $c(r,t) = \frac{1}{r} z(\eta)$, where $\eta = \frac{r}{\sqrt{4Dt}}$.
7. Develop a sample white noise for Brownian excitation acting on 0.05 μm particles in air at room temperature. Evaluate a sample particle trajectory when the there is a uniform air flow velocity of 0.1 m/s in x direction.