ME 437/537 **Final Exam**

1. (40 points) Consider a steady convective-diffusion process with a flow velocity $\mathbf{V} = ay^2\mathbf{i}$ near an absorbing wall. The governing equation is given by

$$ay^2 \frac{\partial C}{\partial x} = D \frac{\partial^2 C}{\partial y^2}$$

where D is the diffusivity and a is a constant. The boundary conditions are:

$$C(0, y) = C_0, \; C(x, \infty) = C_0 \text{ and } C(x,0) = 0.$$

   i. Use a similarity variable $\eta = \frac{y}{2(Dx/a)^{1/4}}$, reduce the governing equation and boundary conditions to the similarity form.
   ii. Evaluate the concentration profile and the deposition velocity to the wall.

2. (35 points) Consider a 12 μm silicon particle that is attached to a silicon wafer in a turbulent air flow with a shear velocity of 2 m/s.

   i. Evaluate the drag, the Saffman lift and the hydrodynamic moment acting on the particle in wall units and in SI units.
   ii. Evaluate the pull-off force as predicted by the JKR model.
   iii. Find the contact radius at zero force and at the separation according to the JKR model.
   iv. Is the particle going to be removed by the rolling mechanism? (Assume $u^+ = y^+$, and for silicon use W_A = 0.0389 J/m^2, E = 1.79x10^{11} N/m^2, and Poisson ratio of 0.27. The kinematic viscosity of air is $\nu = 1.5 \times 10^{-5} m^2/s$)

3. (25 points) Consider a cloud of 12 μm quartz particles with a concentration of 10^5 particles per cm^3.

   i. Find the average absolute number of charge for the equilibrium Boltzmann distribution.
   ii. Determine the number of particles that will carry 5 positive charges. How many will carry no charges in this case?
   iii. Find the mean electrostatic precipitation velocity for a field of 400 Volt/cm for particles with the average absolute charge distribution.
   iv. Find the terminal velocity of these particles and compare with the electrostatic precipitation velocity.

(The density of air is 1.2 kg/m^3, the density ratio of quartz particle to air is 2000, and charge of electron is $e = 1.59 \times 10^{-19}$ Coul.)