Particle Transport, Deposition and Removal

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# Bumpy Particle Removal From Surfaces

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# Outline

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- Static Equilibrium
- Hydrodynamic Forces and Torque
- Adhesion Forces for Bumpy Particles
- Electrostatic Forces
- Capillary Forces
- Rolling and Sliding Removal
- Critical Detachment Shear Velocity



# JKR Adhesion Model

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**Spherical Particles**

**Contact Radius**

$$a^3 = \frac{d}{2K}\left[P + \frac{3W_A \pi d}{2} + \sqrt{3\pi W_A d P + \left(\frac{3\pi W_A d}{2}\right)^2}\right]$$

$$K = \frac{4}{3}\left[\frac{(1-\nu_1^2)}{E_1} + \frac{(1-\nu_2^2)}{E_2}\right]^{-1}$$



# JKR Adhesion Model

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**Pull-Off Force**

$$F_{po}^{JKR} = \frac{3}{4}\pi W_A d$$

**Contact Radius at Separation**

$$a = \left(\frac{3\pi W_A d^2}{8K}\right)^{1/3}$$

# Bumpy Particle

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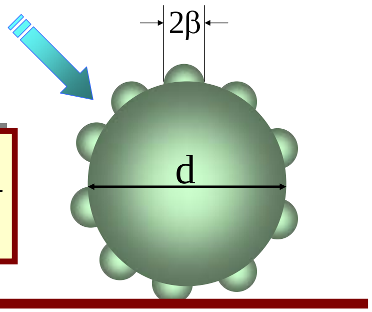
**Schematics of a Bumpy Particle**

$$\beta = \frac{d}{n_u n_b \sqrt{N}}$$






# Pull-Off Force for Bumpy Particles

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**Adhesion Force Per contact Bump**

$$f_{po}^{JKR} = \frac{3}{2}\pi W_A \beta$$

**Total Adhesion Force**

$$F_{ad}^{JKR} = \frac{3}{2}\pi N_c W_A \beta$$



# Capillary Force

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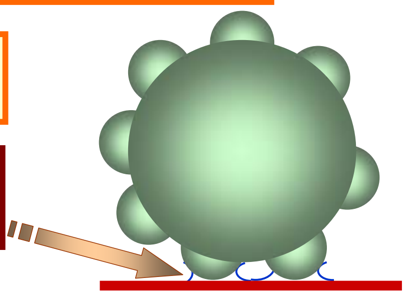
**Capillary Force Per Contact Bump**

$$f_c = 4\pi\sigma\beta$$



**Total Capillary Force**

$$F_c = 4\pi\sigma\beta N_c$$



# Charge Distribution

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**Boltzmann Charge Distribution**

$$f(n) = \frac{e^{-\frac{n^2 e^2}{dkT}}}{\sum_{n=-\infty}^{+\infty} e^{-\frac{n^2 e^2}{dkT}}}$$

**Average Number of Charge**

$$\bar{n} \approx 2.37\sqrt{d}$$

# Number of Charges

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## Boltzmann Charge Distribution

| Diameter $d$ (μm) | Neutral Fraction f(0) | Average Absolute Number of Charges |
|---|---|---|
| 5 | 0.0606 | 5.29 |
| 10 | 0.0428 | 7.46 |
| 15 | 0.0349 | 9.17 |
| 20 | 0.03 | 10.55 |



# Charge Distribution

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## Diffusion Charging

$$n_{diff} = \frac{dkT}{2e^2} \ln\left(1 + \frac{\pi d \bar{c}_i}{2kT} e^2 N_i t\right)$$

## Field Charging

$$n_{field} = \frac{3\varepsilon}{\varepsilon + 2} \frac{Ed^2}{4e}$$



# Number of Charges

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## Diffusion and Field Charging

| Diameter $d$ (μm) | Number of Charges: Diffusion | Field | Combined |
|---|---|---|---|
| 5 | 407 | 4340 | 4747 |
| 10 | 874 | 17361 | 18235 |
| 15 | 1365 | 39062 | 40427 |
| 20 | 1870 | 69444 | 71314 |



# Electrostatic Forces

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## Spherical Particles

$$F_e = \underbrace{qE}_{\text{Coulomb}} - \underbrace{\frac{q^2}{16\pi\varepsilon_o y^2}}_{\text{Image}} + \underbrace{\frac{qEd^3}{16y^3}}_{\text{dielectrophoretic}} - \underbrace{\frac{3}{128}\frac{\pi\varepsilon_o d^6 E^2}{y^4}}_{\text{Polarization}}$$

# Patchy Charge Model

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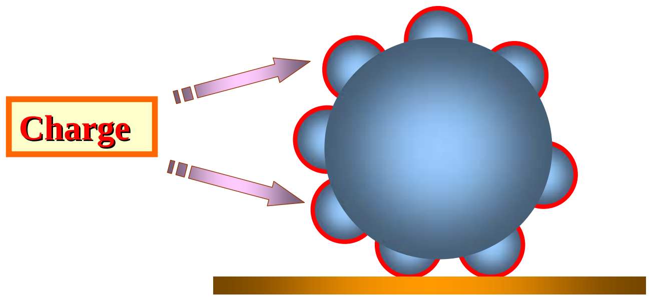




# Electrostatic Forces for Bumpy Particles

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$$F_e = -1.5qE - \frac{q^2}{4\pi\varepsilon_o}\left[\frac{(1-3/N)^2}{d^2} + \frac{[(4n_b^2+1)(3/N)^2}{3\beta^2(4n_b^2+1)^{3/2}}\right] - 72\pi\varepsilon_o\beta^2E^2$$



# Hydrodynamic Forces

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**Drag Force**

$$F_t = \frac{3\pi f\mu d C_d}{C_c} V$$

$$C_d = 1 + 0.15\,Re^{0.687}$$

**Lift Force**

$$F_l = 1.61 d^2 V (\rho\mu)^{1/2} \frac{\frac{dV}{dy}}{\left|\frac{dV}{dy}\right|^{1/2}}$$



# Hydrodynamic Forces

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**Hydrodynamic Torque**

$$M_t = \frac{2\pi\mu f_m d^2 V}{C_c}$$

**Near Wall Peak Velocity**

$$u_M^+ = 1.72\,y^+$$

# Forces

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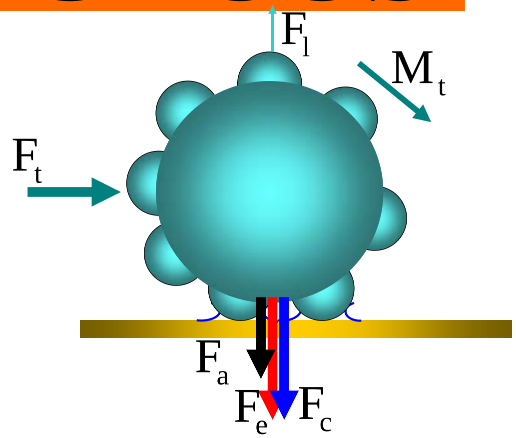


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# Detachment Models

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**Rolling**

$$M_h + F_h \frac{d}{2} \geq (F_{ad} + F_e + F_c) 0.58 n_b \beta$$

**Sliding**

$$F_h \geq k(F_{ad} + F_c + F_e)$$

**Electrostatic**

$$F_{ec} + F_{ed} \geq F_{ad} + F_{ei} + F_{ep} + F_c$$

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# Sublayer Model

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$$u^+ = y^+$$

$$v^+ = -\beta_o y^{+2}, \; y^+ \leq 1.85$$

$$w^+ = 2\beta_o y^+ z^+$$

$$\beta_o = 0.01085$$

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# Electrostatic Forces

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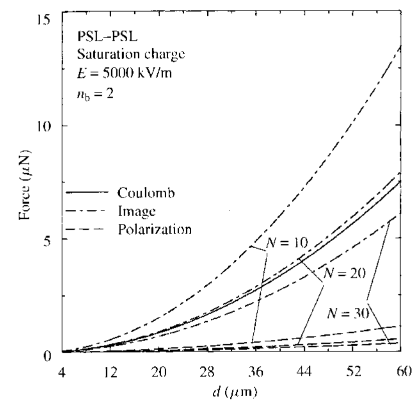
**Saturation Charge**

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# Electrostatic Forces

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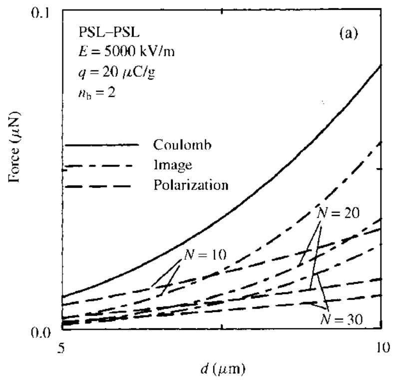
**q=20 μC/g**

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# Critical Shear Velocity

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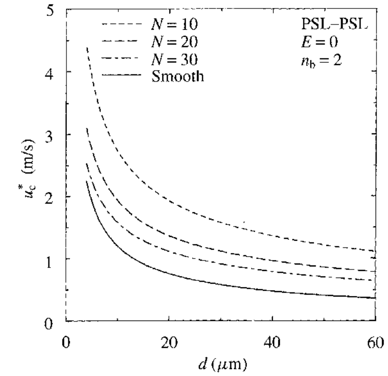
**Neutral Particles**

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# Critical Shear Velocity

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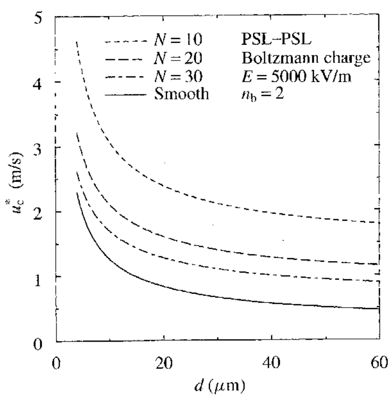
**Boltzmann Charge**

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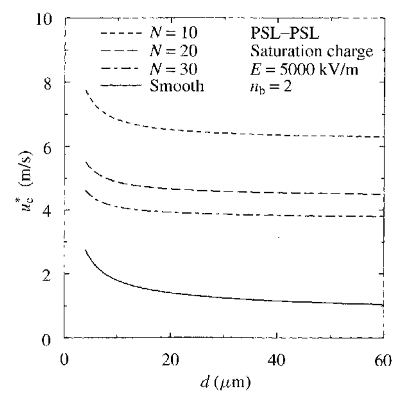
# Critical Shear Velocity

Clarkson University

**Saturation Charge**

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# Critical Shear Velocity

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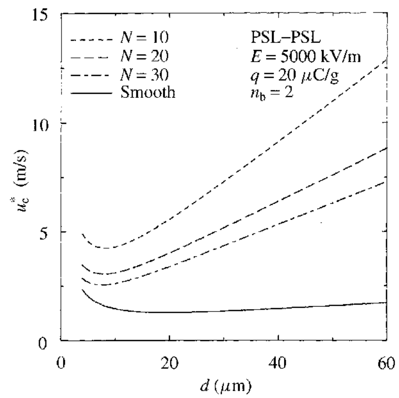
**q=20 μC/g**






# Critical Shear Velocity

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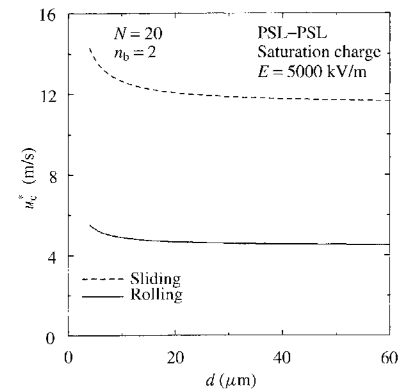
**Saturation Charge**

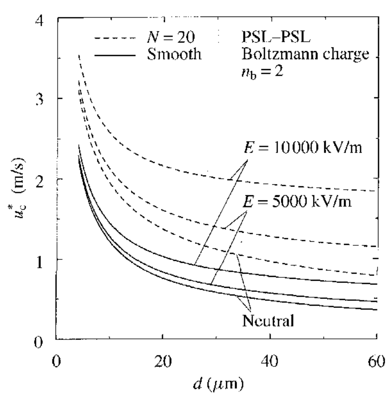





# Critical Shear Velocity

Clarkson University

**Boltzmann Charge**






# Critical Shear Velocity

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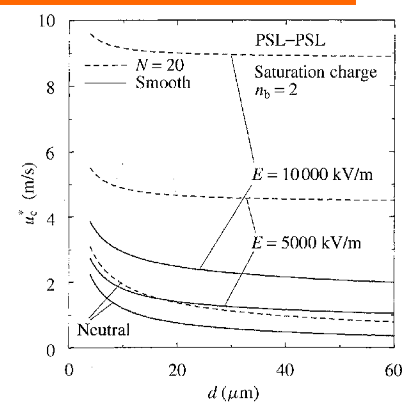
**Saturation Charge**



# Rough Particle

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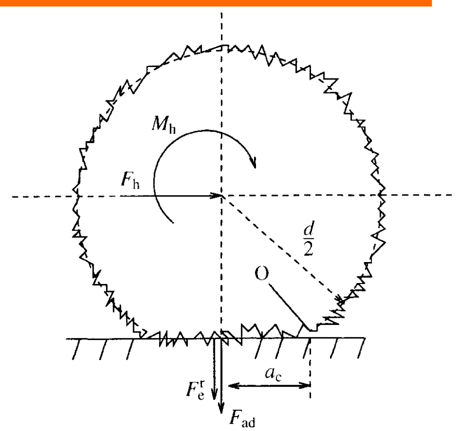




# Rough Particle

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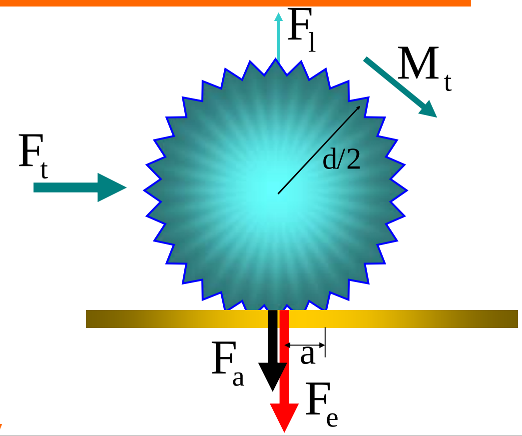




# Rough Particle Contact

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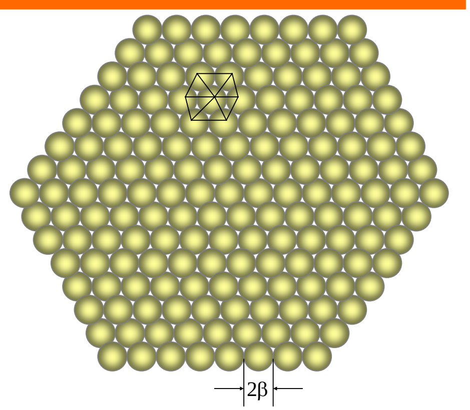




# Electrostatic Forces for Rough Particles

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$$F_e^r = -1.5qE - \frac{q^2}{4\pi\varepsilon_o}\left(\frac{(1-\frac{N_c}{N})^2}{d^2} + \frac{0.13N_c^3}{N^2\beta^2}\right) - 24\pi N_c \beta^2 \varepsilon_o^{\,2} E^2.$$

## Critical Shear Velocity

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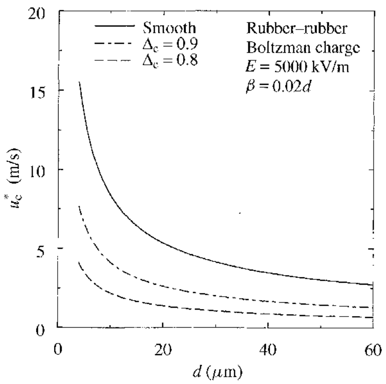
**Boltzmann Charge**

**Rough Particles**

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## Critical Shear Velocity

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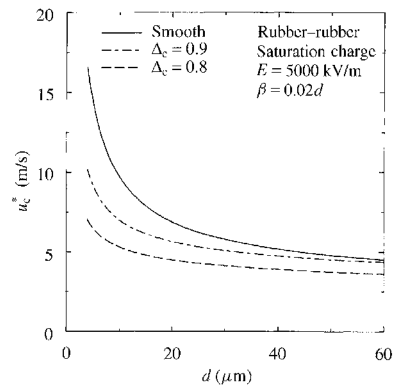
**Saturation Charge**

**Rough Particles**

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## Critical Electric Field

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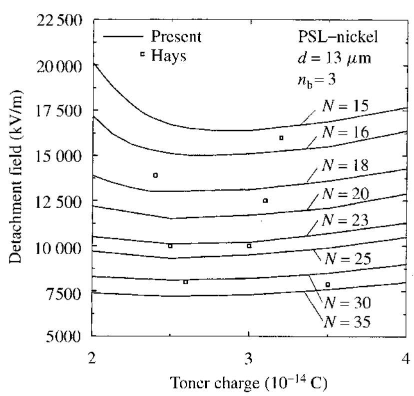


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## Adesion Force

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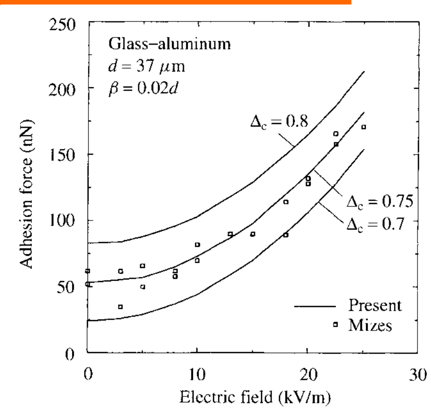
**Rough Particles**

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# Conclusions

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- Rolling detachment is the dominant mechanism for bumpy particle removal in turbulent flows.
- Drag and hydrodynamic torque are dominant for particle detachment from the wall.
- Electrical forces contribute significantly to particle adhesion.
- increasing the number of bumps reduces the adhesion force.
- Patch charge model presents a more realistic picture of surface charge distribution.



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# Thank you!

# Questions?