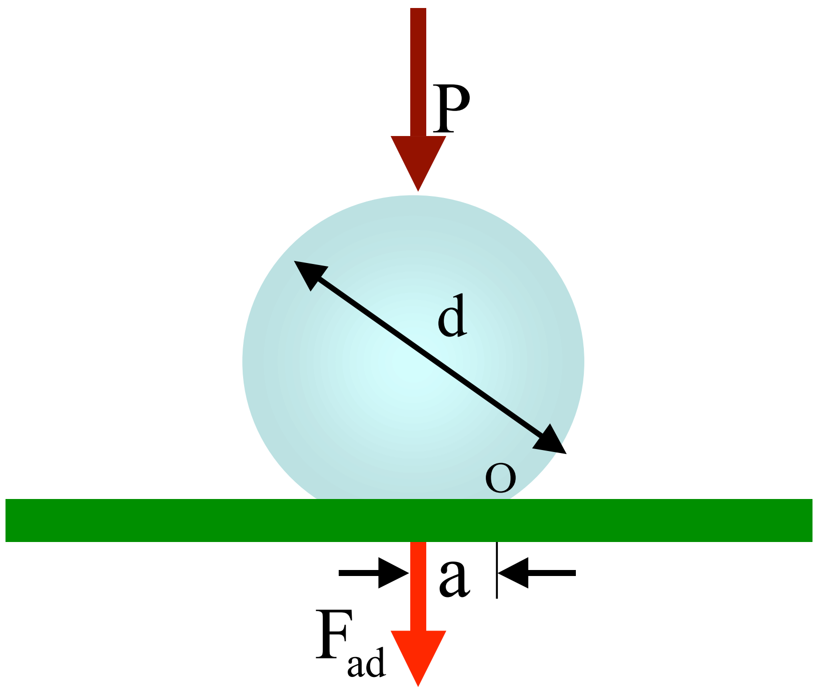
Particle Adhesion and Detachment Models

Figure 1 shows the schematic of a particle of diameter d attached to a flat surface. Here P is the external force exerted on the particle, a is the contact radius and F_{ad} is the adhesion force. The classical Hertz contact theory provides for the elastic deformation of bodies in contact, but neglects the adhesion force. Several models for particle adhesion to flat surfaces were developed in the past that improves the Hertz model by including the effect of adhesion (van der Waals) force.



JKR Model

Johnson-Kandall-Roberts (1971) developed a model (The JKR Model) that included the effect of adhesion force on the deformation of an elastic sphere in contact to an elastic half space. Accordingly, the contact radius is given as

$$a^3 = \frac{d}{2K}\left[P + \frac{3}{2}W_A \pi d + \sqrt{3\pi W_A dP + \left(\frac{3\pi W_A d}{2}\right)^2}\right] \tag{1}$$

Here W_A is the thermodynamic work of adhesion, and K is the composite Young's modulus given as

$$K = \frac{4}{3}\left[\frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2}\right]^{-1} \tag{2}$$

In Equation (2), E is the elastic modulus, ν is the Poisson ratio, and subscript 1 and 2 refer to the materials of the sphere and substrate.

In the absence of surface forces, $W_A = 0$, and Equation (1) reduced to the classical Hertz model. That is

$$a^3 = \frac{dP}{2K} \tag{3}$$

Pull-Off Force

The JKR model predicts that the force needed to remove the particle (the pull-off force) is given as

$$F_{po}^{JKR} = \frac{3}{4}\pi W_A d \tag{4}$$

Contact Radius at Zero Force

The contact radius at zero external force may be obtained by setting P = 0 in Equation (1). That is,

$$a_0 = \left[\frac{3\pi W_A d^2}{2K}\right]^{\frac{1}{3}} \tag{5}$$

Contact Radius at Separation

The contact radius at the separation is obtained by setting $P = -F_{po}^{JKR}$ in Equation (1). The corresponding contact radius is given by

$$a = \left[\frac{3\pi W_A d^2}{8K}\right]^{\frac{1}{3}} = \frac{a_0}{4^{1/3}} \tag{6}$$

DMT Model

Derjaguin-Muller-Toporov (1975) assumed that the Hertz deformation and developed another model that included the effect of adhesion force. According to the DMT model, the poll-off force is given as

$$F_{Po}^{DMT} = \pi W_A d, \qquad (F_{Po}^{DMT} = \frac{4}{3} F_{Po}^{JKR}) \tag{7}$$

<u>*Contact Radius at Zero Force*</u>

The contact radius at zero external force is given as

$$a_0 = \left(\frac{\pi W_A d^2}{2K} \right)^{\frac{1}{3}} \qquad \text{(Hertz contact radius under adhesion force)} \tag{8}$$

<u>*Contact Radius at Separation*</u>

The DMT model predicts that the contact radius at the separation is zero. That is

$$a = 0 \quad \text{(at separation)} \tag{9}$$

Maugis-Pollock

While the JKR and the DMT models assume elastic deformation, there are experimental data that suggests, in many cases, plastic deformation occurs. Maugis-Pollock developed A model that included the plastic deformation effects. Accordingly, the relationship between the contact radius and external force is given as

$$P + \pi W_A d = \pi a^2 H \tag{10}$$

where H is hardness and

$$H = 3Y, \tag{11}$$

with Y being the yield strength.

Note that variations of contact radius with particle diameter at equilibrium, that is in the absence of external force, for elastic and plastic deformation are different. That is

$$a_0 \sim d^{\frac{2}{3}} \quad \text{(elastic)}, \qquad a_0 \sim d^{\frac{1}{2}} \quad \text{(plastic)} \tag{12}$$

Thermodynamic Work of Adhesion

The thermodynamic work of adhesion (surface energy per unit area) is given as

$$W_A = \frac{A}{12\pi z_0^2} \tag{13}$$

where A is the Hamaker constant and z_o is the minimum separation distance.

Non-Dimensional Forms

Nondimensional form of the relationship between contact radius and the external force and the corresponding moment are described in the section.

JKR Model

Equation (1) in nondimensional form may be restated as

$$a^{*3} = 1 - P^* + \sqrt{1 - 2P^*} \tag{14}$$

where the nondimensional external force and contact radius are defined as

$$P^* = -\frac{P}{\frac{3}{2}\pi W_A d}, \qquad a^* = \frac{a}{\left(\frac{3\pi W_A d^2}{4K}\right)^{\frac{1}{3}}} \tag{15}$$

Variation of the nondimensional contact radius with the nondimensional force is shown in Figure 2. Note that for $P^* = 0$, Equation (14) and Figure 2 shows that $a_0^* = 1.26$.

The corresponding resistance moment about point O in Figure 1 as a function of nondimensional force is given as

$$M^{*JKR} = P^* a^* = P^*(1 - P^* + \sqrt{1 - 2P^*})^{1/3} \tag{16}$$

Figure 3 shows the variation of the resistance moment as predicted by the JKR model. The corresponding maximum resistance moment then is given by

$$M_{max}^{*JKR} = 0.42 \tag{17}$$

and

$$P^*_{max} = F^{*JKR}_{po} = \frac{F^{JKR}}{\frac{3\pi}{2} W_A d} = 0.5 \tag{18}$$

The resistance moment at P^* is $M^{*JKR} = 0.397$. Also

$$P^*_{max} a^*_0 = 0.63 \tag{19}$$

DMT

For DMT Model, the approximate expression for the contact radius is given as

$$a^3 \approx \frac{d}{2K}\left(P + \pi W_A d\right) \tag{20}$$

or

$$a^{*3} = \left(\frac{a}{3\pi W_A d^2 / 4K}\right)^3 = -P^* + \frac{2}{3} \tag{21}$$

Variation of the nondimensional contact radius with the nondimensional force as predicted by the DMT model is shown in Figure 2 and is compared with the JKR model. Note that for $P^* = 0$, Equation (21) and Figure 2 shows that $a^*_0 = 0.874$.

The corresponding resistance moment as a function of nondimensional force as predicted by the DMT model is given as

$$M^{*DMT} = P^*(2/3 - P^*)^{1/3} \tag{22}$$

The variation of the resistance moment as predicted by the DMT model is also shown in Figure 3. The corresponding maximum resistance moment is

$$M^{*DMT}_{max} = 0.28 \tag{23}$$

Note also that the maximum force (the poll-off force) is given by

$$P^*_{max} = F^{*DMT}_{po} = \frac{F^{DMT}}{\frac{3\pi}{2} W_A d} = \frac{2}{3} \tag{24}$$

and

$$P^*_{max} a^*_0 = 0.58 \tag{25}$$

Comparing Equations (17) and (23) shows that the JKR model predicts a larger resistance moment. That is

$$M^{*JKR}_{max} = 0.42 = 1.5 M^{*DMT}_{max}, \qquad (M^{DMT}_{max} = 0.28) \tag{26}$$

The resistance moment predicted by the JKR and the DMT models in dimensional form are given as

$$M^{JKR}_{max} = 2.63 \frac{W_A^{\frac{4}{3}} d^{\frac{5}{3}}}{K^{\frac{1}{3}}}, \quad M^{DMT}_{max} = 1.83 \frac{W_A^{\frac{4}{3}} d^{\frac{5}{3}}}{K^{\frac{1}{3}}} \tag{27}$$

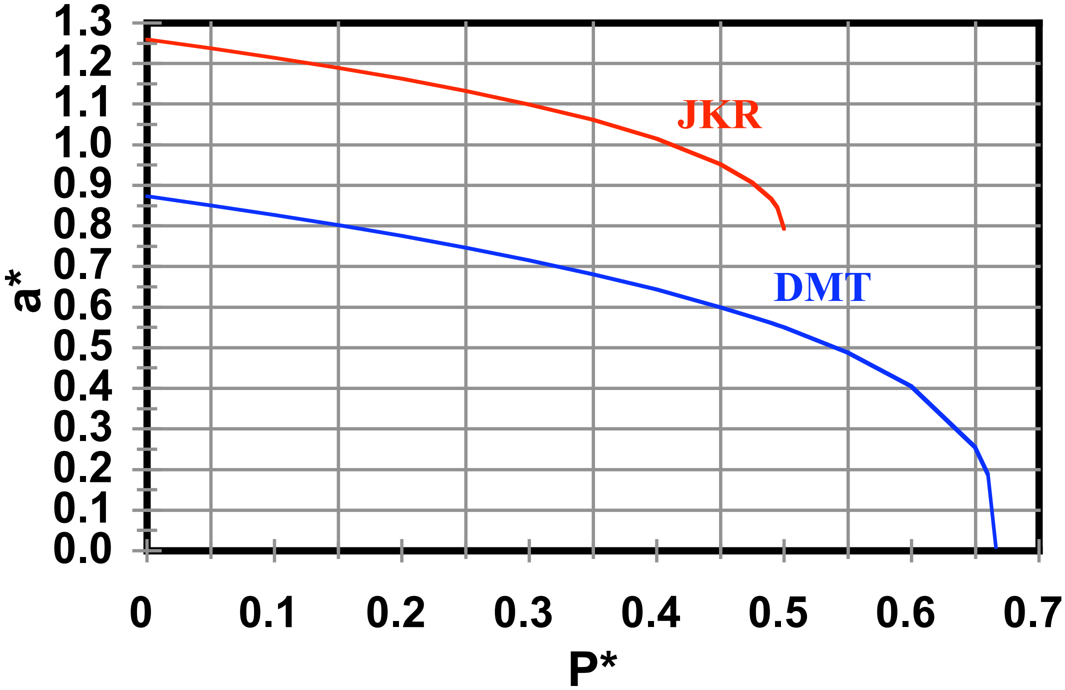


Figure 2. Variations of contact radius with the exerted force.

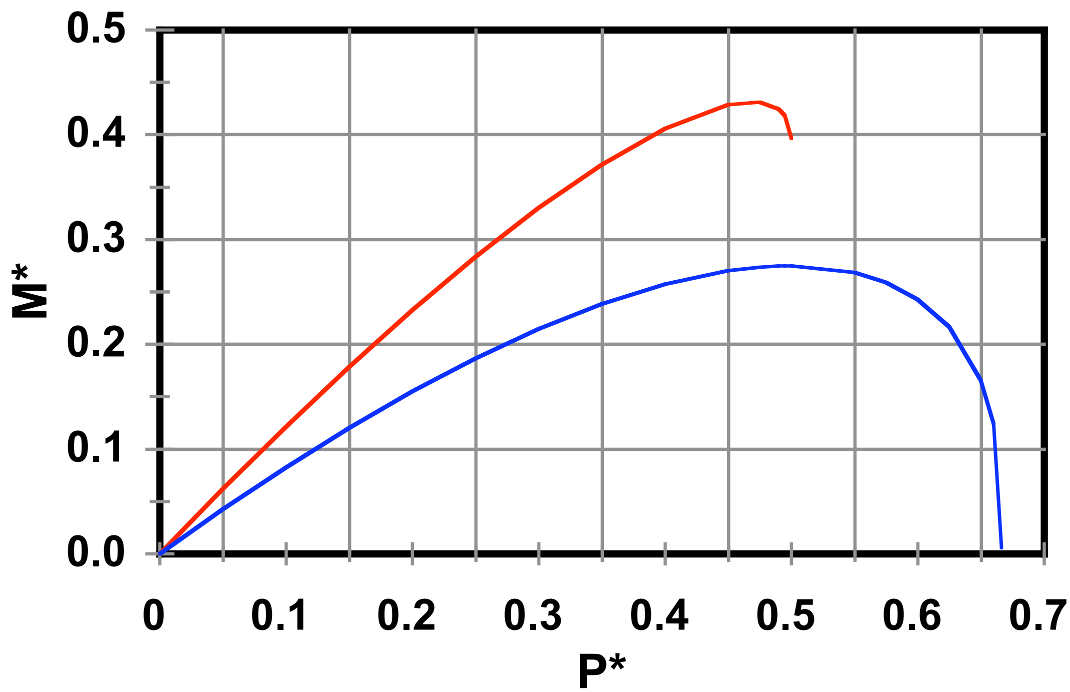


Figure 3. Variations of resistance moment with the exerted force.