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London-van der Waals Force

Goodarz Ahmadi
Department of Mechanical and Aeronautical Engineering
Clarkson University
Potsdam, NY 13699-5725

Outline

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- Features of van der Waals Force
- Sphere Near a Plane
- Cylinder near a Flat Plate
- Interfaces
- Hamaker Constants
- Hamaker Constants for Dissimilar Materials

London-van der Waals Force Features

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- Generally Attractive
- Short Range
- Origin in Atomic Dipole

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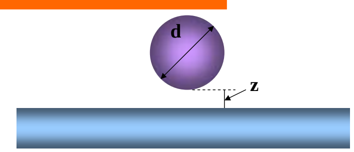
Interaction Energy



$$\phi = -\frac{A}{12}\left[\frac{1}{x} + \frac{1}{1+x} + 2\ln\frac{x}{1+x}\right]$$

$$x = \frac{z}{d}$$

A=Hamaker Const.

Range of Application

$$z \to 0 \quad \Rightarrow \quad \phi \approx -\frac{Ad}{12z}$$

$$z \le \frac{Ad}{12kT} \sim 0.2d$$

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Particle	Surface	$F_v \times 10^8$ N (air)	$\frac{F_v}{3\pi\mu dU}$	$F_v \times 10^8$ N (water)	$\frac{F_v}{3\pi\mu dU}$
Poly-styrene	Poly-stynne	1.2-1.8	70-100	0.2	12
Si	Si	13.6-14.4	800-850	7	410
Cu	Cu	17	1000	9.8	580
Ag	Ag	18	1060	15.5	910

$z_o = 4\overset{o}{A}$ $U = 1\,m/s$

Comparison of Forces

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	van der Waals	Surface Tension	Added Mass	Drag/Lift	Basset
Diameter d (μm)	$F_v \sim A_{123}\frac{d}{12z_o^2}$	$F_{st} \sim 2\pi\gamma d$	$F_{am} \sim \rho d^3 \frac{dV}{dt}$	$F_D \sim \rho^f d^2 V^2$	$F_B \sim \frac{\mu d^2 V}{\sqrt{\nu t}}$
0.2	3×10^{-8}	9×10^{-5}	10^{-18}	10^{-12}	4×10^{-15}
2	3×10^{-7}	9×10^{-4}	10^{-15}	10^{-10}	4×10^{-13}
20	3×10^{-6}	9×10^{-3}	10^{-12}	10^{-8}	4×10^{-11}

F_v

Comparison of Forces

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	van der Waals	Surface Tension	Added Mass	Drag/Lift	Basset
Diameter d (μm)	$F_v \sim A_{123}\frac{d}{12z_o^2}$	$F_{st} \sim 2\pi\gamma d$	$F_{am} \sim \rho d^3 \frac{dV}{dt}$	$F_D \sim \rho^f d^2 V^2$	$F_B \sim \frac{\mu d^2 V}{\sqrt{\nu t}}$
0.2	2×10^{-9}		8×10^{-16}	8×10^{-10}	10^{-12}
2	2×10^{-8}		8×10^{-13}	8×10^{-8}	10^{-10}
20	2×10^{-7}		8×10^{-10}	8×10^{-6}	10^{-8}

F_v

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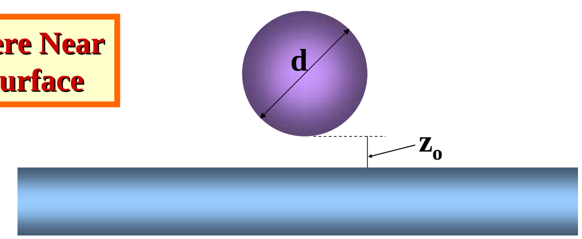
Sphere Near a Surface



$$F = \frac{A_{132} d}{12 z_o^2}$$

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Cylinder Near a Surface

$$\frac{F}{\text{length}} = \frac{A_{132} d^{1/2}}{16 z_0^2}$$

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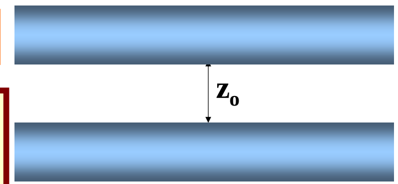
Planar Surfaces



$$\frac{F}{\text{area}} = \frac{A_{132}}{6\pi z_0^3}$$

Surface Energy

$$\phi = \frac{A_{132}}{12\pi z_0^2}$$

Hamaker Constants for Dissimilar Materials

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Two Materials

$$A_{12} \approx \sqrt{A_{11} A_{22}}$$

$$A_{12} = \frac{2 A_{11} A_{22}}{A_{11} + A_{22}}$$

Three Materials

$$A_{132} = A_{12} + A_{33} - A_{13} - A_{23}$$

$$A_{131} = A_{11} + A_{33} - 2A_{13} = \frac{(A_{11} - A_{33})^2}{A_{11} + A_{33}} \simeq (\sqrt{A_{11}} - \sqrt{A_{33}})^2$$

Table of Hamaker Constants $A/10^{-20}$ J

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Materials	Vacuum	Water
Polystyrene	7.9	1.3
Gold	40	30
Silver	50	40
Al_2O_3	16.75	4.44
Copper	40	30
Water	4.0	-

Table of Hamaker Constants $h\bar{\omega}_{131}$ (eV)

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Combinations	Water	Polystyrene
Au-Cu	6.41	5.93
Au-Diamond	6.11	5.45
Au-Si	5.32	4.70
Au-Ge	6.50	5.93
Au-MgO	1.99	1.25

Surface Energy Between Particles

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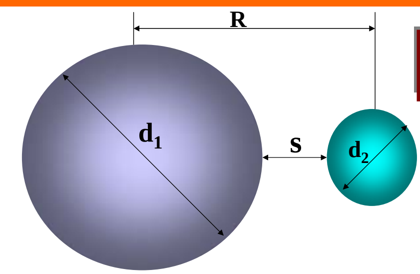

$$R = \frac{d_1 + d_2}{2} + s$$

$$\Phi = -\frac{A}{6}\left[\frac{d_1 d_2/2}{R^2 - (\frac{d_1+d_2}{2})^2} + \frac{d_1 d_2/2}{R^2 - (\frac{d_1-d_2}{2})^2} + \ln\frac{R^2 - (\frac{d_1+d_2}{2})^2}{R^2 - (\frac{d_1-d_2}{2})^2}\right]$$

Surface Energy Between Particles

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For Equal Sizes

$$d_1 = d_2 = d$$

$$r = d + s$$

$$\Phi = -\frac{A}{6}\left[\frac{d^2}{2r^2} + \frac{d^2}{2(r^2 - d^2)} + \ln(1 - \frac{d^2}{r^2})\right]$$

$$A_{121} = A_{11} + A_{22} - A_{12} \approx \sqrt{(A_{11} - A_{22})}$$

Conclusions

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- **van der Waals force is very large at short distances**
- **Short range force**
- **Can be computed for particles, cylinders and plane interfaces**