Brownian Motion

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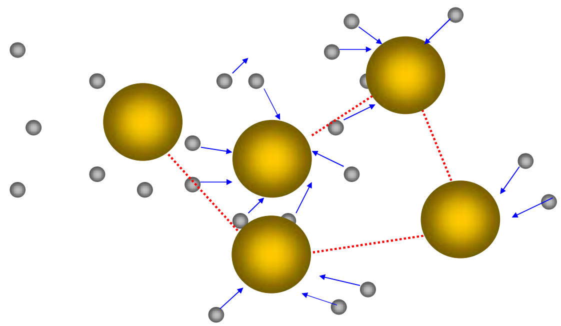
Outline

- Introduction to Aerosols
- Drag Forces
- Cunningham Corrections
- Lift Forces
- **Brownian Motion**
- Particle Deposition Mechanisms
- Gravitational Sedimentation
- Aerosol Coagulation

Brownian Motion



A Particle under Random Molecular Impact

Brownian Motion

Langevin Equation →

$$\frac{du}{dt}+\beta u = n(t)$$

$$\beta = 3\pi\mu d / C_c m = 1/\tau$$

N(t) = White Noise

Spectral Intensity →

$$S_{nn} = \frac{2kT\beta}{\pi m}$$

Brownian Motion - Solving a Stochastic Equation

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Autocorrelation Function

$$R(\tau) = \overline{u(t+\tau)u(t)}$$

Fourier Transform

$$R_{uu}(\tau) = \frac{1}{2}\int_{-\infty}^{+\infty} e^{i\omega\tau} S_{uu}(\omega)d\omega$$

$$S_{uu}(\omega) = \frac{1}{\pi}\int_{-\infty}^{+\infty} e^{-i\omega\tau} R_{uu}(\tau)d\tau$$

Brownian Motion - Solving a Stochastic Equation

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$$S_{uu}(\omega) = |H(\omega)|^2 S_{nn}(\omega)$$

System Function

$$H(\omega) = \frac{1}{i\omega + \beta}$$

Response Power Spectrum

$$S_{uu}(\omega) = \frac{2kT\beta/\pi m}{\omega^2 + \beta^2}$$

Autocorrelation

$$R_{uu}(\tau) = \frac{kT}{m}e^{-\beta|\tau|}$$

Brownian Motion

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Mass Diffusivity

$$D = \frac{1}{2}\frac{d}{dt}\overline{x^2}(t)$$

Position

$$x(t) = \int_0^t u(t_1)dt_1$$

Variance

$$\overline{x^2}(t) = \int_0^t\int_0^t R_{uu}(\tau_1 - \tau_2)d\tau_1 d\tau_2$$

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Variance

$$\overline{x^2}(t) = 2\int_0^t (t-\tau)R_{uu}(\tau)d\tau$$

Diffusivity

$$D = \int_0^\infty R_{uu}(\tau)d\tau$$

$$D = \frac{kT}{\beta m} = \frac{kTC_c}{3\pi\mu d}$$

Brownian Motion -Markov Process Fokker-Plank Approach

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Fokker-Planck Equation

$$\frac{\partial f}{\partial t} - \frac{\partial}{\partial u}(\beta u f) = \frac{kT\beta}{m}\frac{\partial^2 f}{\partial u^2}$$

Probability Density →

$$f = \frac{1}{\sqrt{2\pi kT/m}} e^{-\frac{mu^2}{2kT}}$$

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Brownian Motion in a Force Field

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Langevin Equation →

$$\ddot{x} + \beta\dot{x} - \frac{F(x)}{m} = n(t)$$

$$F(x) = -\frac{\partial V(x)}{\partial x}$$

Fokker-Planck Equation

$$\frac{\partial f}{\partial t} = -\frac{\partial(\dot{x}f)}{\partial x} + \frac{\partial}{\partial \dot{x}}[(\beta\dot{x} - \frac{1}{m}F(x))f] + \frac{kT\beta}{m}\frac{\partial^2 f}{\partial \dot{x}^2}$$

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Brownian Motion in a Force Field

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Probability Density

$$f = C_0 \exp\{-\frac{m}{kT}[\frac{\dot{x}^2}{2} - \int_0^x \frac{F(x_1)dx}{m}]\}$$

$$f = C_0 \exp\{-\frac{1}{kT}[\frac{m\dot{x}^2}{2} + V(x)]\}$$

Gravitational Field →

$$f = C_0 e^{-\frac{m\dot{x}^2}{2kT}} e^{-\frac{mg(x-x_0)}{kT}}$$

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Computer Simulation Procedure

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White Noise

$$\overline{n(t)} = 0$$

$$\overline{n(t_1)n(t_2)} = 2\pi S_{nn}\delta(t_1 - t_2)$$

- **Choose a time step ($\Delta t << \tau$)**
- **Generate a sequence of uniform random numbers ($0 < U < 1$)**
- **Transform to Gaussian random numbers.**

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Computer Simulation Procedure

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- $$G_1 = \sqrt{-2\ln U_1}\cos 2\pi U_2$$
  $$G_2 = \sqrt{-2\ln U_1}\sin 2\pi U_2$$
- **Amplitude of the Brownian force is given by** $n(t_i) = G_i\sqrt{\frac{\pi S_{nn}}{\Delta t}}$
- **The generated sample of Brownian force need to be shifted by** $U\Delta t$

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Sample Simulated White Noise Process

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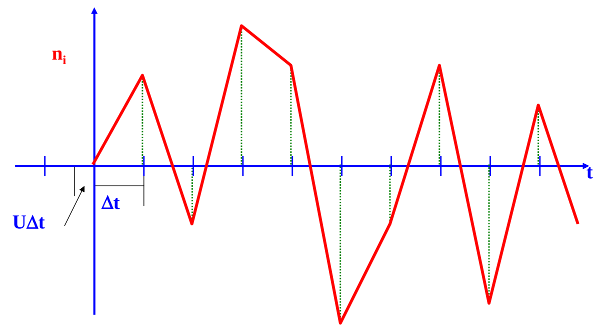

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Sample Simulated White Noise Process

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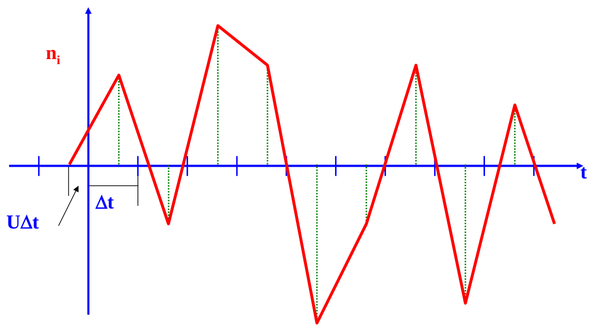

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Particle Dispersion and Deposition in Viscous Sublayer

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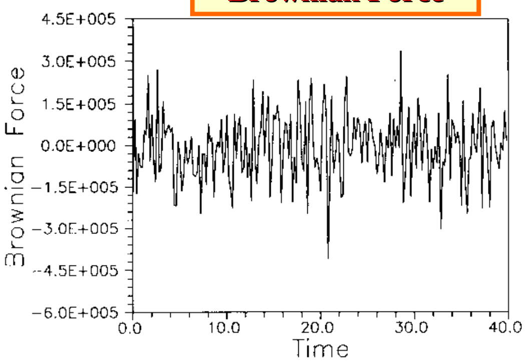
Brownian Force



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Particle Dispersion in Viscous Sublayer

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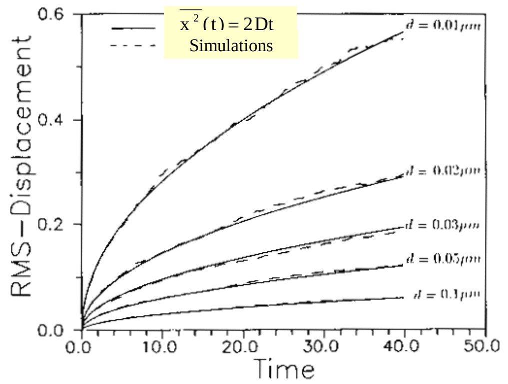

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Particle Dispersion in Viscous Sublayer

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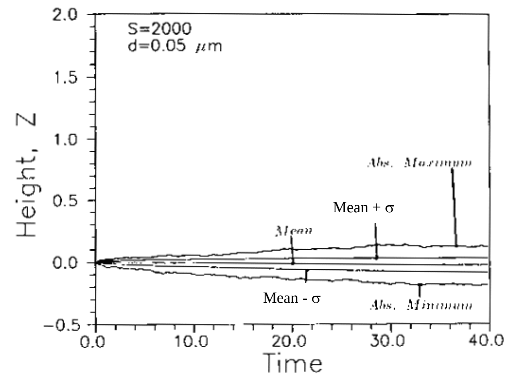

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Particle Dispersion in Viscous Sublayer

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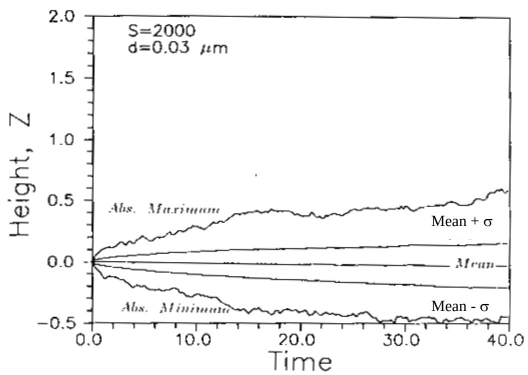

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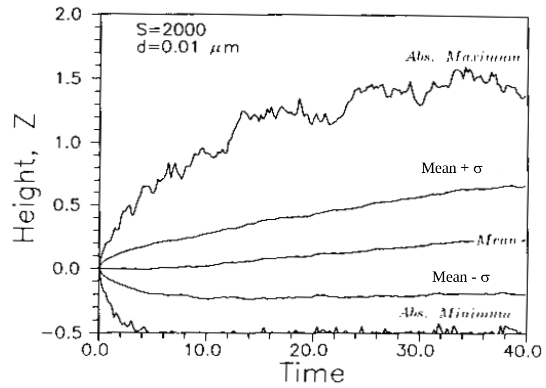

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Particle Dispersion in Viscous Sublayer

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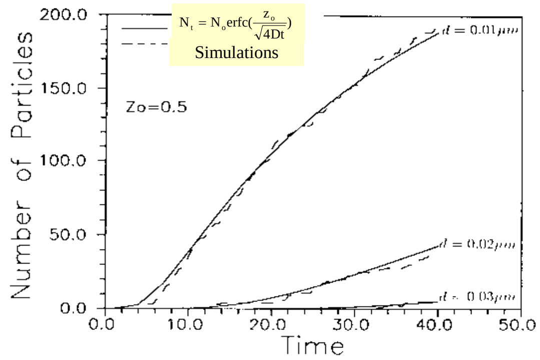

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Java Applet for Brownian Motion

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Equation of Motion

$$\frac{d\mathbf{u}^p}{dt} = \frac{1}{\tau}(\mathbf{u}^f - \mathbf{u}^p) + \mathbf{g} + \mathbf{n}(t)$$

Variance ⇒ $\sigma_y^2(t) = 2Dt$

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Particle Dispersion -Java Applet for particle trajectory analysis

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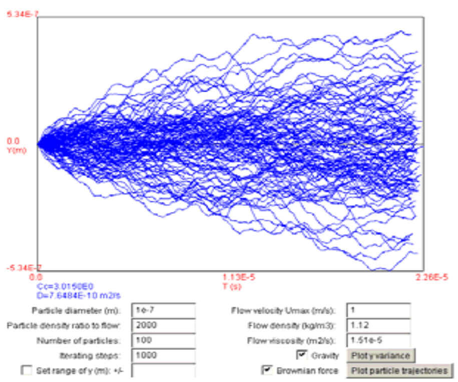

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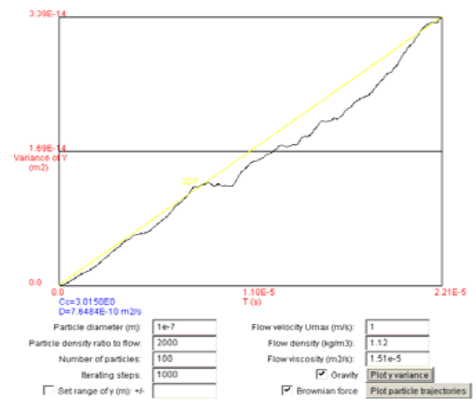

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