

# Aerosols Charging and Their Kinetics

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# Aerosols Charging

## Outline

- ▶ Electrostatics
- ▶ Particle Charging
- ▶ Charged Particle Kinetic

# Aerosols Charging and Their Kinetics

Most aerosol particles carry some electrical charges

Coulomb Force



$$F_E = qE$$

$$q = ne$$

Electric Charge

$$e = 1.6 \times 10^{-19} \text{ coul}$$

$$e = 4.8 \times 10^{-10} \text{ statcoul}$$

Particle Mobility

$$qE = 3\pi\mu Ud / C_c$$



$$u = Z^p = \frac{qC_c}{3\pi\mu d}$$

# Particle Charging

Boltzmann Equilibrium Charge Distribution



$$f(n) = \frac{\exp\{-n^2 e^2 / dkT\}}{\sum_{n=-\infty}^{\infty} \exp\{-n^2 e^2 / dkT\}}$$

$$f(n) = \sqrt{\frac{e^2}{dkT\pi}} \exp\left\{-\frac{n^2 e^2}{dkT}\right\}$$

$$d > 0.02\mu$$

$$f(n) = \frac{0.24}{\sqrt{d\pi}} \exp\left\{-\frac{0.05n^2}{d}\right\}$$

$$d > 0.02\mu$$

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## Average Number of Charge

$$\bar{n} = \sum_{-\infty}^{\infty} |n| f(n) \approx \int_{-\infty}^{\infty} |n| f(n) dn \approx \sqrt{\frac{dkT}{\pi e^2}}$$

$d > 0.02 \mu$

$\bar{n} \approx 2.36\sqrt{d}, d(\mu m)$

**Point Charge** →  $E = \frac{\gamma q}{4\pi r^2}$       $\gamma = 4\pi / \epsilon$      cgs

**Air** →  $\epsilon = 1$       $\gamma = 4\pi$       $\gamma = \frac{1}{\epsilon_0 \epsilon}$      MKS

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# Coulomb's Law Clarkson University

$$F = q'E = \frac{\gamma q'q}{4\pi r^2}$$

$$\gamma = \frac{1}{\epsilon_0 \epsilon}$$

## Permittivity

$$\epsilon_0 = 8.859 \times 10^{-12} \frac{\text{amp} - \text{sec}}{\text{volt} - \text{meter}}$$

## Coulomb's Law

$$F = \frac{q'q}{\epsilon r^2} (9 \times 10^9)$$

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# Field Charging Clarkson University

$$n = \left[ \frac{\pi e Z_i n_{i\infty} t}{\pi e Z_i n_{i\infty} t + 1} \right] \left( 1 + \frac{2(\epsilon_p - 1)}{\epsilon_p + 2} \right) \frac{Ed^2}{4e} \quad \text{cgs}$$

$$n_{\infty} = \left[ 1 + \frac{2(\epsilon_p - 1)}{\epsilon_p + 2} \right] \frac{Ed^2}{4e} \quad \text{as } t \rightarrow \infty \quad \text{cgs}$$

$$\epsilon_p = 4.3 \text{ for Quartz}$$

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# Diffusion Charging Clarkson University

$$n = \frac{dkT}{2e^2} \ln \left[ 1 + \left( \frac{2\pi}{m_i kT} \right)^{1/2} n_{i\infty} d e^2 t \right]$$

$$n_{i\infty} t \approx 10^8 \text{ ion sec/cm}^3$$

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# Field and Diffusion Charging

Diameter μm	Number of Units of Charge		
	Diffusion	Field	Combined
0.01	0.276	0.0007	0.277
0.02	0.672	0.0027	0.675
0.03	1	0.0062	1.12
0.05	2.1	0.02	2.12
0.1	5	0.07	5.1
0.5	32	2	34
1	69	7	76
2	149	28	177
3	234	63	297
5	414	174	588
10	889	694	1583
20	1901	2778	4679
50	5162	17361	22523
100	10954	69583	80537
200	23121	277778	300899
300	35767	625000	660767

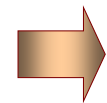
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# Particle Drift in an Electric Field

**Equation of Motion** →

$$m \frac{d\mathbf{u}^p}{dt} = \mathbf{F}_D + \mathbf{F}_G + \mathbf{F}_E$$



$$\tau \frac{d\mathbf{u}^p}{dt} = \mathbf{u}^f - \mathbf{u}^p + \tau \mathbf{g} - \mathbf{E}q \frac{\tau}{m}$$



$$\tau \frac{d\mathbf{u}^p}{dt} + \mathbf{u}^p = \mathbf{u}_o - \mathbf{E}q \frac{\tau}{m}$$

$$\mathbf{u}_o = \mathbf{u}^f + \tau \mathbf{g}$$

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# Particle Drift in an Electric Field

For  $\mathbf{u}_o \parallel \mathbf{E}$  →

$$\tau \frac{d\mathbf{u}^p}{dt} + \mathbf{u}^p = \mathbf{u}_o - \Gamma \mathbf{u}^p$$

$$\mathbf{u}^p = \frac{\mathbf{u}_o}{1 + \Gamma \tau}$$

$$\Gamma = \frac{Eq\tau}{m\epsilon_0}$$

For  $|\Gamma| \gg 1$ , neglecting inertia →

$$\mathbf{u}^p = -\Gamma \mathbf{u}_o$$

$$\mathbf{u}^p = -\mathbf{E}q \frac{\tau}{m}$$

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# Electrical Forces

$$F_e = qE - \frac{q^2}{16\pi\epsilon_0 y^2} + \frac{qEd^3}{16y^3} - \frac{3}{128} \frac{\pi\epsilon_0 d^6 E^2}{y^4}$$

Coulomb Force
Image Force
Polarization Force

Dipole Interactions

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