ME529 **Homework 1**

1. If a space S consists of n elements, show that the total number of its subsets are 2^n.

2. If $a \cap b \neq 0$, show $P(a \cup b) = P(a) + P(b) - P(a \cap b)$.

3. Show that $P(a \cap b \cap c) = P(a \mid b \cap c) P(b \mid c) P(c)$.

4. Is it possible that two events are independent and mutually exclusive?

5. The probability that an electron is emitted from a substance in an interval (t_1, t_2), $t_2 > t_1 > 0$ is given by
$$P\left\{t_1 \le t \le t_2\right\} = e^{-\beta t_1} - e^{-\beta t_2} \qquad \beta = const$$
Find $P\left\{t_0 \le t \le t_0 + \tau \mid t \ge t_0\right\}$

6. Two fair dice are rolled 10 times, find the probability p that "seven" will show at least once.

7. A fair coin is tossed n=900 times. Find the probability p_0 that the number of heads will be between 420 and 465.

8. We place at random n particles in m>n boxes. Find probability p that the particles will be in n pre-selected boxes, one in each box. Solve the problem for the following three cases:
    i. Particles are distinguishable.
    ii. Particles are not distinguishable.
    iii. Particles are not distinguishable and only one particle can be placed in each box.

[Hint. Consult Chapter 1 and problems in Chapter 3 of Papoulis.]