ME529 **Hints for Homework 1**

1. **If a space S consists n elements, show that the total number of its subsets is 2^n.**

Hint:

$S=\{\zeta_1,\zeta_2,.....\zeta_6\}$. The subsets are:

$$\underbrace{\{O\}}_{\binom{n}{0}},\ \underbrace{\{\zeta_1\},\{\zeta_2\},....\{\zeta_n\}}_{\binom{n}{1}},\ \underbrace{\{\zeta_1,\zeta_2\},\{\zeta_1,\zeta_3\},....\{\zeta_n,\zeta_{n-1}\}}_{\binom{n}{2}}\cdots\cdots \underbrace{S}_{\binom{n}{n}}$$

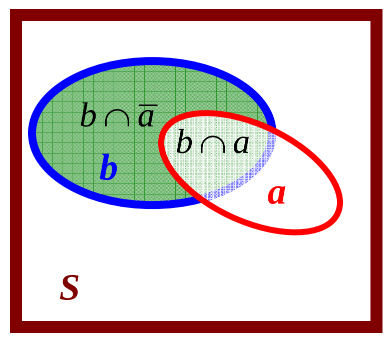
2. **If $a\cap b\neq 0$, show** $P(a\cup b)=P(a)+P(b)-P(a\cap b)$

Hint:

$$a\cup b=a\cup(\bar{a}\cap b)$$

$$P(a\cup b)=P(a\cup(\bar{a}\cap b))=P(a)+P(\bar{a}\cap b).$$

$$b=(b\cap a)\cup(b\cap\bar{a})$$



3. **Show that** $P(a\cap b\cap c)=P(a\,|\,b\cap c)P(b\,|\,c)P(c)$

Hint:

$$P(b\,|\,c)=\frac{P(b\cap c)}{P(c)},\ \text{thus,}\quad P(b\,|\,c)P(c)=P(b\cap c)$$

4. **Is it possible that two events are independent and mutually exclusive?**

Hint:

Independent $\Rightarrow$ $P(a\cap b)=P(a)P(b)$

Mutually exclusive events $\Rightarrow$ $P(a\cap b)=0$

5. **The probability that an electron is emitted from a substance in an interval (t_1, t_2), $t_2>t_1>0$ is given by**

$$P\left\{t_1 \le t \le t_2\right\} = e^{-\beta t_1} - e^{-\beta t_2} \qquad \beta = const$$

Find $P\left\{t_0 \le t \le t_0 + \tau \mid t \ge t_0\right\}$

Hint: Use the definition of conditional probability.

6. **Two fair dice are rolled 10 times, find the probability p that "seven" will show at least once.**

Hint:

There are six possibilities for "seven" to show, namely, {1+6, 2+5, 3+4, 4+3, 2+5, 6+1}. The probability that seven occurs in one trial is 1/6 and probability that seven does not occur in on trial is 5/6,

7. **A fair coin is tossed n=900 times. Find the probability that the number of heads will be between 420 and 465.**

Hint:

Use the Demoivre-Laplace approximation,

$$P_n\left(k_1 \le k \le k_2\right) = \sum_{k=k_1}^{k_2} \binom{n}{k} p^k q^{n-k} \approx erf\left(\frac{k_2 - np}{\sqrt{npq}}\right) - erf\left(\frac{k_1 - np}{\sqrt{npq}}\right).$$

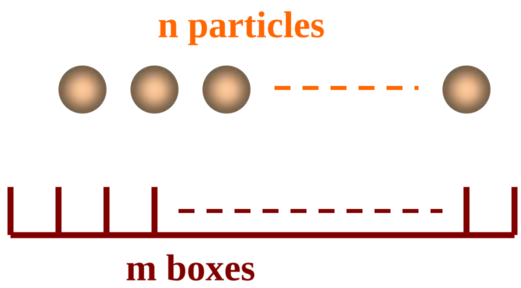
For n = 900, p=q=1/2, k_1 = 420, k_2 =465

8. **We place at random n particles in *m>n* boxes. Find probability p that the particles will be in *n* pre-selected boxes, one in each box. Solve the problem for the following three cases:**
    - i) **Particles are distinguishable.**
    - ii) **Particles are not distinguishable.**
    - iii) **Particles are not distinguishable and only one particle can be placed in each box.**

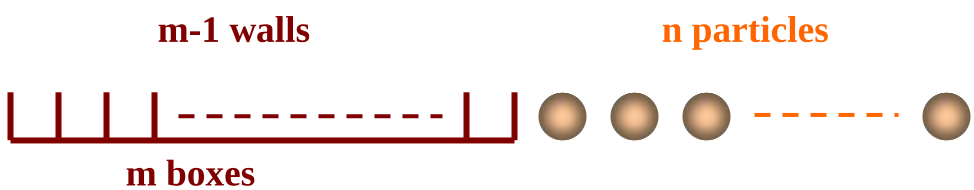
Hint:

This is the central problem in statistical mechanics.

i) Particles are distinguishable. (Maxwell-Botzmann)



ii) Particles are not distinguishable. (Bose-Einstein)



Total number of permutations of *m-1* wall and *n* particle =*(n+m-1)!*

Particles are indistinguishable; their number of permutation=*n!*
Walls are indistinguishable; their number of permutation=*(m-1)!*

iii) Particles are indistinguishable and only one particle can be placed in each box. (Fermi-Dirac)

Total number of ways that n distinguishable particles can be placed in m boxes =

$$m\times(m-1)\times(m.-2)..\times(m-n+1)=\frac{m!}{(m-n)!}$$