ME529 **FINAL EXAM** **SAMPLE EXAM**

Throughout the following problems $n_i(t)$ (or $n(t)$) denote Gaussian white noise processes with

$$E\{n_i(t)\}=0,\quad R_{n_i n_j}(t_1,t_2)=2D_{ij}\delta(t_1-t_2),\quad D_{ij}=0 \text{ for } i\neq j.$$

1. Consider the following system of random equations:

$$\frac{dX}{dt}+Y(t)=n(t),\qquad \frac{dY}{dt}+X(t)=n(t).$$

a) Determine $S_{XX}(\omega)$, $S_{YY}(\omega)$. b) $R_{XX}(\tau)$, and $E\{X^2\}$. c) $E\{X(t)^6\}$ and $E\{X(t)^5\}$.

2. Suppose Z and W are two independent random variables with

$$f_Z(z)=\begin{cases}\frac{1}{2} & \text{for } 0\le z\le 2\\ 0 & \text{Otherwise}\end{cases},\quad \text{and}\quad P(W=0)=\frac{1}{4},\quad P(W=1)=\frac{1}{2},\ P(W=2)=\frac{1}{4}$$

We define two random processes $X(t)=\delta(Z-t)$ and $Y(t)=U(W-t)$. For $0\le t\le 1$,
a) Determine $E\{X(t)]$, $E\{Y(t)\}$, and $R_{XY}(t_1,t_2)$. b) Find $P\{-1\le Z\le 1\cup -1\le W\le 1\}$

3. Let

$$Z(t)=\int_0^t \tau X(\tau)d\tau$$

with $R_{XX}(t_1,t_2)=|t_1-t_2|\,\mathrm{Min}\{t_1,t_2\}$. Find $E\{Z^2(t)\}$.

4. Consider the following random differential equation.

$$\frac{dX}{dt}+2X=Yn_1(t),$$
$$\frac{dY}{dt}+Y^3=X\,n_2(t).$$

Set up the Fokker-Planck equation.

Table of Fourier Exponential Transform Pair

$$\bar{f}(\omega) = \int_{-\infty}^{+\infty} e^{-i\omega t} f(t)\,dt\,, \qquad f(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{i\omega t}\bar{f}(\omega)\,d\omega$$

$f(t)$	$\bar{f}(\omega)$
$f_1(t)e^{-i\omega_o t}$	$\bar{f}_1(\omega+\omega_o)$
$f_1(t+t_o)$	$e^{i\omega t_o}\bar{f}(\omega)$
$f_1(t) * f_2(t) = \int_{-\infty}^{+\infty} f_1(\tau) f_2(t-\tau)\,d\tau$	$\bar{f}_1(\omega)\bar{f}_2(\omega)$
$\delta(t-t_o)$	$e^{-i\omega t_o}$
$e^{i\omega_o t}$	$2\pi\delta(\omega-\omega_o)$
$e^{-\alpha\|t\|}$	$\frac{2\alpha}{\omega^2+\alpha^2}$
$\cos\omega_o t$	$\pi[\delta(\omega-\omega_o)+\delta(\omega+\omega_o)]$
$e^{-\alpha\|t\|}\cos\beta t$	$\frac{2\alpha(\omega^2+\alpha^2+\beta^2)}{(\omega^2-\beta^2-\alpha^2)^2+4\alpha^2\omega^2}$
$e^{-\alpha\|t\|}\left[\cos\beta t+\frac{\alpha}{\beta}\sin\beta\|t\|\right]$	$\frac{4\alpha(\alpha^2+\beta^2)}{(\omega^2-\beta^2-\alpha^2)^2+4\alpha^2\omega^2}$
$e^{-\alpha^2t^2}\cos\beta t$	$\frac{\sqrt{\pi}}{2\alpha}\left[\exp\left\{-\frac{(\omega+\beta)^2}{4\alpha^2}\right\}+\exp\left\{-\frac{(\omega-\beta)^2}{4\alpha^2}\right\}\right]$
$e^{-\alpha^2t^2}$	$\frac{\sqrt{\pi}}{\alpha}\exp\left\{-\frac{\omega^2}{4\alpha^2}\right\}$
$\frac{d^n}{dt^n}\delta(t)$	$(i\omega)^n$
$J_0(t)$	$\begin{cases}\frac{2}{\sqrt{1-\omega^2}} & \|\omega\|<1 \\ 0 & \text{elsewhere}\end{cases}$
$\frac{\sin\omega_o t}{\pi t}$	$\begin{cases}1 & \|\omega\|\le\omega_o \\ 0 & \|\omega\|>\omega_o\end{cases}$
$\begin{cases}1-\left\|\frac{t}{T}\right\| & \|t\|\le T \\ 0 & \|t\|>T\end{cases}$	$4\frac{\sin^2(\omega T/2)}{T\omega^2}$