ME529 **FINAL EXAM** **Sample Final-1**

Throughout the following problems $n_i(t)$ (or n(t)) denote Gaussian white noise processes with

$$E\{n_i(t)\} = 0,\quad R_{n_i n_j}(t_1, t_2) = 2D_{ij}\delta(t_1 - t_2),\quad D_{ij} = 0 \text{ for } i \neq j.$$

1. Suppose Z and W are two independent random variables with

$$f_Z(z) = \begin{cases} \frac{1}{4} & \text{for } -3 \le z \le 1 \\ 0 & \text{Otherwise} \end{cases}$$

$$P(W = -1) = \frac{1}{3},\quad P(W = 1) = \frac{2}{3}$$

   We define two random processes X(t) = δ(Z+W+t) and Y(t) = (Z-Wt). For $0 \le t \le 1$,
   a) Determine E{X(t)], E{Y(t)}, and $R_{XY}(t_1,t_2)$. b) Find $P\{-1 \le Z \le 2 \cup -2 \le W \le 0\}$

2. Consider the following system of random equations:

$$\frac{d^2X}{dt^2} + \frac{dX}{dt} + 2Y + \int_{-\infty}^{+\infty} (Y(\tau) + 2\frac{dY(\tau)}{d\tau})e^{-|t-\tau|}\,d\tau = \frac{dn}{dt}$$

$$\frac{dX}{dt} + X(t) + \int_{-\infty}^{+\infty} [\frac{d^2Y(\tau)}{d\tau^2} + Y(\tau)]\, e^{-2(t-\tau)^2}\,d\tau = n(t)$$

   Determine $S_{XX}(\omega)$, $S_{YY}(\omega)$, $R_{XX}(\tau)$, and $E(X^2)$.

3. Consider the following random differential equation.

$$\frac{d^2X}{dt^2} + n_1(t)\frac{dX}{dt} + n_2(t)X = 0.$$

   a) Set up the Fokker-Planck equation.
   b) Drive the general moment equation, and the equations governing E{X}, $E\{X^2\}$, $E\{\dot{X}\}$, $E\{\dot{X}^2\}$, and $E\{X\dot{X}\}$.

4. Given that

$$\frac{dX}{dt} + [1 + n_1(t) + n_2(t)]\,X = 0$$

   a) Obtain the Fokker-Planck equation.
   b) Derive the general moment equation and the governing equations for E{X}, $E\{X^2\}$ and $E(e^X\}$.