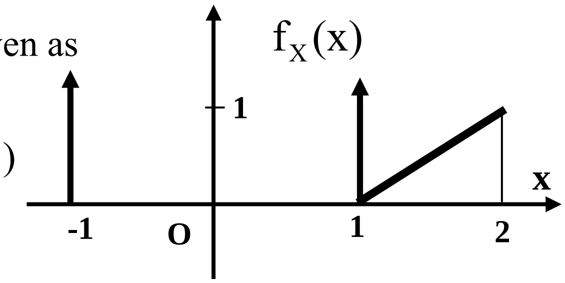
ME 529 **EXAM 1**

1) The probability density of a random variable X is given as



$$f_x(x) = (x-1)u(x-1)u(2-x) + \frac{1}{4}\delta(x-1) + \frac{1}{4}\delta(x+1)$$

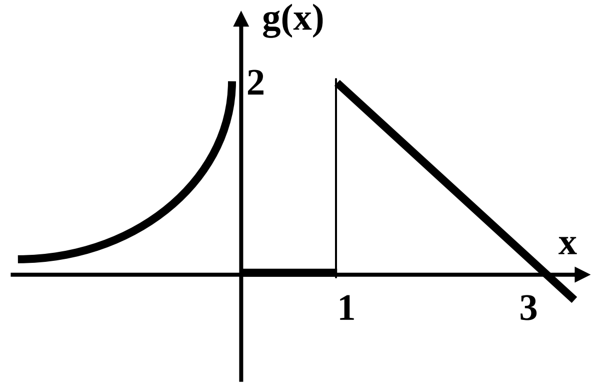
a) Evaluate mean and variance of X.

b) Evaluate the Characteristic function of X, $\varphi_x(\omega)$.

c) Evaluate P{-0.5< X <1.5}.

d) Evaluate E(X|X≤1.5).

2) For



$$Y = g(X) = \begin{cases} -1/X & for \quad X \le 0 \\ 0 & for \quad 0 < X \le 1 \\ 3 - X & for \quad X > 1 \end{cases}$$

a) Determine the probability density of Y in terms of density of X for any $f_x(x)$.

b) For $f_x = e^{-x}u(x)$, determine $f_y(y)$.

c) Find E{Y}

3) Given $f_{xy}(x,y)$ and $Z = \frac{X}{Y}$,

a) Find $f_{ZY}(z,y)$ and $f_z(z)$.

b) Evaluate $f_{ZY}(z,y)$ and $f_z(z)$ for $f_{xy}(x,y) = e^{-(x+y)}u(x)u(y)$.

c) Evaluate $E\{ZY^2\}$.

d) Find $P\{-2 < X \le 2 \cap -1 < Y \le 1)$.

{Hint $\int_0^\infty x^m e^{-ax}dx = \frac{m!}{a^{m+1}}$ and $\int P(x)e^{ax}dx = \frac{e^{ax}}{a}(P(x) - \frac{P'(x)}{a} + \frac{P''(x)}{a^2} - ...)$}.