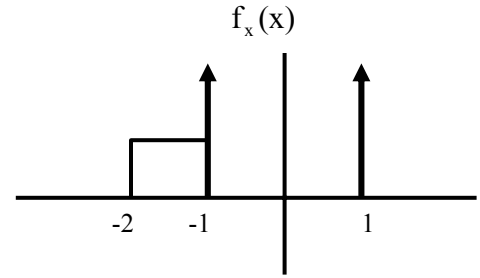


1) The probability density of a random variable X is given as

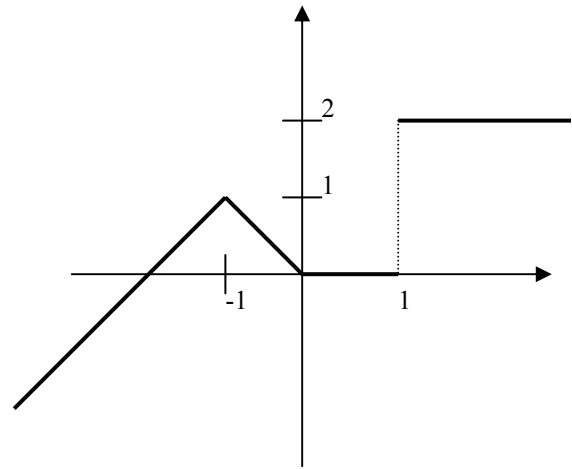
$$f_x(x) = \frac{1}{3} \{ [u(x+2) - u(x+1)] + \delta(x-1) + \delta(x+1) \}$$



- a) Evaluate mean and variance of X
- b) Evaluate $\varphi_x(\omega)$
- c) Evaluate $P\{-1.5 < X < -0.5\}$.

2) For

$$Y = g(X) = \begin{cases} X + 2 & \text{for } X \leq -1 \\ -X & \text{for } -1 \leq X \leq 0 \\ 0 & \text{for } 0 \leq X < 1 \\ 2 & \text{for } X \geq 1 \end{cases}$$



- a) Determine the probability density of Y in terms of density of X.
- b) For $f_x = e^{-x}u(x)$, determine $f_y(y)$.
- c) Find $E\{Y\}$

3) Given $f_{xy}(x, y)$ and $Z = XY^3$,

- a) Find $f_{zy}(z, y)$ and $f_z(z)$.
- b) Evaluate $f_{zy}(z, y)$ and $f_z(z)$ for $f_{xy}(x, y) = y^3 e^{-xy^3} u(x) u(y) u(1-y)$.
- c) Evaluate $E(Z)$ and $E(X)$
- d) Find $P\{-2 < X \leq \infty \cap -1 < Y \leq 0.5\}$.

{Hint $\int_0^{\infty} x^m e^{-ax} dx = \frac{m!}{a^{m+1}}$ }.