ME 529 - Stochastics

Equivalent Linearization Technique

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Equivalent Linearization Technique

Outline

- Equivalent Linear System
- Mean-Square Error
- Equivalent Parameters
- Stationary Response

Equivalent Linearization Technique

Equivalent Linear System

$$\ddot{X} + g(X,\dot{X}) = f(t)$$

$$\ddot{X} + \beta_e \dot{X} + \omega_e^2 X = f(t)$$

Mean-Square Error

$$E\{(error)^2\} = E\{e^2\} = E\{(\beta_e \dot{X} + \omega_e^2 X - g(X,\dot{X}))^2\}$$

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Equivalent Parameters Minimize M-S Error

$$\frac{\partial}{\partial \beta_e} E\{e^2\} = 0 = 2E\{\dot{X}[\beta_e \dot{X} + \omega_e^2 X - g(X,\dot{X})]\}$$

$$\frac{\partial}{\partial (\omega_e^2)} E\{e^2\} = 0 = 2E\{X[\beta_e \dot{X} + \omega_e^2 X - g(X,\dot{X})]\}$$

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Equivalent Parameters Minimize M-S Error

$$\omega_e^2 = \frac{E\{\dot{X}^2\}E\{Xg\} - E\{X\dot{X}\}E\{\dot{X}g\}}{E\{X^2\}E\{\dot{X}^2\} - \left(E\{X\dot{X}\}\right)^2}$$

$$\beta_e = \frac{E\{X^2\}E\{\dot{X}g\} - E\{X\dot{X}\}E\{Xg\}}{E\{X^2\}E\{\dot{X}^2\} - \left(E\{X\dot{X}\}\right)^2}$$

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Stationary Response Analysis

$$E\{X\dot{X}\} = 0$$

$$\omega_e^2 = \frac{E\{Xg\}}{E\{X^2\}} \qquad \beta e = \frac{E\{\dot{X}g\}}{E\{\dot{X}^2\}}$$

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Stationary Response Analysis

Noting that

$$\frac{\partial^2 E\{e^2\}}{\partial(\omega_e^2)^2} = 2E\{X^2\} \qquad \frac{\partial^2 E\{X^2\}}{\partial \beta_e^2} = 2E\{\dot{X}^2\} \qquad \frac{\partial^2 E\{e^2\}}{\partial \beta_e \partial(\omega_e^2)} = 2E\{X\dot{X}\}$$

$$E\{X^2\}E\{\dot{X}^2\} - E\{X\dot{X}\} \geq 0$$

∴ Mean-Square Error is Minimized

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Gaussian Processes

$$\omega_e^2 = E\left\{\frac{\partial g}{\partial X}\right\}$$

$$\beta e = E\left\{\frac{\partial g}{\partial \dot{X}}\right\}$$

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