# ME 529 - Stochastics

# Stochastic Processes

Goodarz Ahmadi

Department of Mechanical and Aeronautical Engineering
Clarkson University
Potsdam, NY 13699-5725



# Outline

- Definition of Stochastic Processes
- 1st Order Density and Distribution
- 2nd Order Density and Distribution
- Conditional Density
- Expected Value and Autocorrelation
- Auto-covariance and Variance
- Examples of Physical Stochastic Processes

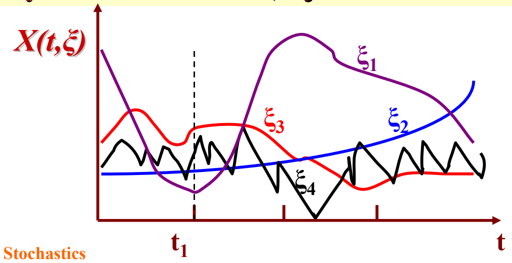


# Stochastic Processes

Given a random experiment $\Im$: (S, F, P), to each $\xi$ we assign time function $X(t,\xi)$ $t \in (0,T)$

***Stochastic process***

family of time functions, $\xi \in S$.



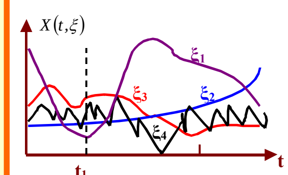



# Stochastic Processes

**Meaning of $X(t,\xi)$**

- $t,\xi$ vary: family of time functions
- $t$ vary: single time functions
- $\xi$ vary: random variable
- $t,\xi$ fixed: single number

Note: $X_1 = X(t_1,\xi)$ ... $X_n = X(t_n,\xi)$ are n random variables



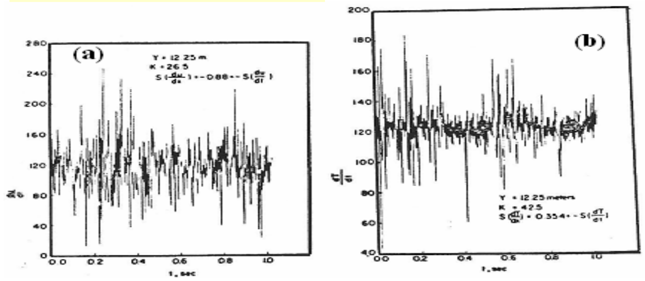

Example of a stochastic process. Sample space has four outcomes.

# Sample Stochastic Processes

Clarkson University

Derivative of Air Velocity

Derivative of Air Temp Over Ocean






# Stochastic Processes

Clarkson University

***Alternative definition*: Stochastic process is a family of random variables $X(t_1)$, $X(t_2)$,… for $t \in (0,T)$.**

**i. Discrete – set is finite/countably infinite**

**ii. Continuous – set is non-countably infinite**



# Density & Distribution Functions

Clarkson University

**1st Order Density & Distribution Functions**

$$F(x,t) = P\{X(t) \le x\}$$

$$f(x;t) = \frac{\partial F(x;t)}{\partial x}$$

**2nd Order Density & Distribution Functions**

$$F(x_1,x_2;t_1,t_2) = P\{X(t_1) \le x_1 \cap X(t_2) \le x_2\}$$

$$f(x_1,x_2;t_1,t_2) = \frac{\partial^2 F(x_1,x_2;t_1,t_2)}{\partial x_1 \partial x_2}$$



# Stochastic Processes- Stratnovich Definition

Clarkson University

**Stratnovich 1st Order Density Function**

$$f(x;t) = E\{\delta(X(t) - x)\}$$

**Stratnovich 2nd Order Density Function**

$$f(x_1,x_2;t_1,t_2) = E\{\delta(X(t_1) - x_1)\delta(X(t_2) - x_2)\}$$

**Properties**

$$F(x_1,\infty;t_1,t_2) = F(x_1;t_1)$$

$$f(x_1;t_1) = \int_{-\infty}^{+\infty} f(x_1,x_2;t_1,t_2)\,dx_2$$

# Statisticers of Stochastic Processes

Clarkson University

**Conditional Density**

$$f(x_1;t \mid X(t_2)=x_2)=\frac{f(x_1,x_2;t_1,t_2)}{f(x_2,t_2)}$$

**Mean Value**

$$\eta(t)=E\{X(t,\xi)\}=\int_{-\infty}^{+\infty} x f(x;t)\,dx$$

**Autocorrelation**

$$R(t_1,t_2)=E\{X(t_1)X(t_2)\}=\int_{-\infty}^{+\infty} x_1x_2 f(x_1,x_2;t_1,t_2)\,dx_1dx_2$$

**Autocovariance**

$$C(t_1,t_2)=E\{[X(t_1)-\eta(t_1)][X(t_2)-\eta(t_2)]\}$$

**or**

$$C(t_1,t_2)=R(t_1,t_2)-\eta(t_1)\eta(t_2)$$

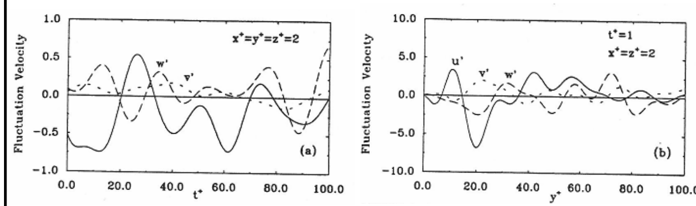
**Variance**

$$\sigma_{X(t)}^2(t)=C(t,t)=R(t,t)-\eta^2(t)$$



# Examples of Stochastic Processes

Clarkson University

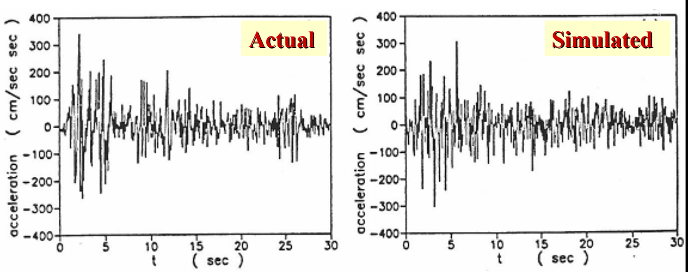



**Sample time & space variations of fluctuation velocity components in turbulent near wall flows**



# Examples of Stochastic Processes

Clarkson University




**Accelerograms of 1940 El Centro Earthquakes**



# Examples of Stochastic Processes

Clarkson University

**STS-41 Z Lift-Off Acceleration**

**Original**

**Simulated**