Mean - Square Estimation

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Mean-Square Estimation

Outline

- Estimation of Random Variable with Constant
- Nonlinear Mean-Square Estimation
- Linear Mean-Square Estimation
- Several Random Variable Estimates
- Jointly Normal Random Variables

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Estimation of Random Variable by Constant

$E\{(X-\alpha)^2\}$

$I = E[(X-\alpha)^2] = E\{X^2\} - 2\alpha E\{X\} + \alpha^2$

Minimizing Error I: $\frac{\partial I}{\partial \alpha} = 0$ $\alpha = E\{X\}$

Nonlinear Mean-Square Estimate of Y by g(X)

Minimizing $E\{[Y-g(X)]^2\}$: $g(X) = E\{Y \mid X\}$

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Linear Mean-Square Estimation g(X) = aX + b

$a = \frac{r\sigma_Y}{\sigma_X}$ $b = E\{Y\} - aE\{X\}$ $e_m = \sigma_Y^2(1-r^2)$ $r = E\frac{\{(X-\eta_X)(Y-\eta_Y)\}}{\sigma_X \sigma_Y}$

(e_m = minimum error)

If $\eta_X = \eta_Y = 0$, $b = 0$, $a = \frac{E\{XY\}}{E\{X^2\}}$, $e_m = E\{Y^2\} - E\{(aX)^2\}$

Note that *a* minimizes $E\{(Y-aX)^2\}$, **i.e.** $E\{(Y-aX)X\} = 0$.
⇨ ***X* is orthogonal to** $Y - aX$ **and** $e_m = E\{(Y-aX)Y\}$

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***Theorem*: If X and Y are jointly normal, nonlinear and linear mean-square estimation of Y in terms of X leads to identical solution:**

$E\{Y \mid X\} = aX$

$a = E\frac{\{XY\}}{E\{X^2\}}$

Several Random Variable M-S Estimation

Minimizing $E\{[X_0 - g(X_1,\dots,X_n)]^2\}$

$X_0 = g(X_1,\dots,X_n) = E\{X_0 \mid X_1,\dots,X_n\}$

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Linear Mean-Square Estimation

$g = a_1 X_1 + \dots + a_n X_n = \sum_i a_i X_i$

$E\left\{\left(X_0 - \sum_i a_i X_i\right) X_j\right\} = 0$

$R_{0j} = \sum_i R_{ji} a_i$

$j = 1,\dots,n$

Jointly Normal Random Variables X_0, X_1, ... X_n

Minimizing $E\{X_0 \mid X_1,\dots,X_n\} = \sum_i a_i X_i$

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Concluding Remarks

- Estimation of Random Variable with Constant
- Nonlinear Mean-Square Estimation
- Linear Mean-Square Estimation
- Several Random Variable Estimates
- Jointly Normal Random Variables

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Thank you!

Questions?

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