ME 529 - Stochastics Clarkson Several Random Variables

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Several Random Variables Clarkson

Outline

- > Several Random Variables
- > Joint Distribution, Density Functions
- > Independent Random Variables
- > Expected Value
- **≻** Covariance
- > Joint Characteristic Function

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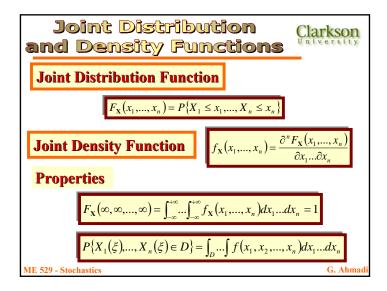
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Given a probability experiment \Im : (S, F, P), a random vector $X(\xi) = (X_1(\xi), X_2(\xi), ..., X_n(\xi))$ is defined as a mapping of the probability space unto a point of the n-dimensional Euclidean space R^n . That is $X(\xi)$ is defined by a certain rule for every $\xi \in S$.

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Independent Random Variables

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The random variables $X_1, X_2, ..., X_n$ are said to be independent if the events $\{X_1 \le x_1\}, ..., \{X_n \le x_n\}$ are independent for any $x_1, ..., x_n$.

$$F_{\mathbf{X}}(x_1, x_2, ..., x_n) = F_1(x_1)F_2(x_2)...F_n(x_n)$$

$$f_x(x_1, x_2,..., x_n) = f_1(x_1)f_2(x_2)...f_n(x_n)$$

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Expected Value

 $E\{g(X_1,...,X_n)\} = \int_{-\infty}^{+\infty} ... \int_{-\infty}^{+\infty} g(x_1,...,x_n) f_X(x_1,...,x_n) dx_1...dx_n$

Covariance

 $c_{ij} = E\{(X_i - \eta_i)(X_j - \eta_j)\} = E\{X_i X_j\} - \eta_i \eta_j$

 $\eta_i = E\{X_i\}$

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Joint Characteristic Function

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 $\Phi_{\mathbf{X}}(\omega_1, ..., \omega_n) = E\left\{e^{i(\omega_1 X_1 + ... + \omega_n X_n)}\right\} = E\left\{e^{i\boldsymbol{\omega} \cdot \mathbf{X}}\right\}$

Characteristic and Density Function of Fourier Transform Pair

$$\Phi_{\mathbf{X}}(\mathbf{\omega}) = \int_{-\infty}^{+\infty} ... \int_{-\infty}^{+\infty} e^{i\mathbf{\omega} \cdot \mathbf{x}} f_{\mathbf{X}}(\mathbf{x}) dx_1 ... dx_n$$

$$f_{\mathbf{X}}(\mathbf{x}) = \frac{1}{(2\pi)^n} \int_{-\infty}^{+\infty} \dots \int_{-\infty}^{+\infty} e^{-i\mathbf{\omega} \cdot \mathbf{x}} \Phi_{\mathbf{x}}(\mathbf{\omega}) d\omega_1 \dots d\omega_n$$

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If X₁, X₂, ..., X_n are independent random variables

$$\Phi(\omega_1,...,\omega_n) = \Phi_1(\omega_1)...\Phi_n(\omega_n)$$

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Concluding Remarks

- > Several Random Variables
- > Joint Distribution, Density Functions
- > Independent Random Variables
- > Expected Value
- > Covariance
- > Joint Characteristic Function

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