ME 529 - Stochastics

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Transformation of Random Variables

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Transformation of Random Variables

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Outline

- Transformation of a Random Variable
- Fundamental Transformation Theorem
- Justification

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Transformation of One Random Variable

Let $Y(\xi) = g[X(\xi)]$

$$F_Y(y) = P\{Y(\xi) \le y\} = P\{g(X(\xi)) \le y\}$$

Then

$$f_Y(y) = \frac{dF_y(y)}{dy}$$

Transformation Theorem

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Given $f_X(x)$ and $Y = g(X)$,

Then

$$f_Y(y) = \sum_{i=1}^{n} \frac{f_X(x_i(y))}{|g'(x_i(y))|}$$

where $x_i = g^{-1}(y)$ are n real roots for a given y.
If for some value of y there is no real root, then

$$f_Y(y) = 0$$

Transformation Theorem

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Justification

By definition $f_Y(y)dy = P\{y < Y \le y + dy\}$

Suppose for a given *y* there are *n* roots $y_i = g(x_i)$

Thus $f_Y(y)dy = P\{x_1 < X < x_1 + dx_1 \cup \ldots \cup x_n < X \le x_n + dx_n\}$

or $f_Y(y)dy = \sum_{i=1}^{n} f_X(x_i)|dx_i|$ $\Rightarrow$ $f_Y(y) = \sum_{i=1}^{n} \frac{f_X(x_i)}{\frac{dy}{|dx_i|}} = \sum_{i=1}^{n} \frac{f_X(x_i)}{|g'(x_i)|}$

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Concluding Remarks

- Transformation of a Random Variable
- Fundamental Transformation Theorem
- Justification

Transformation

Thank you!

Questions?