ME 529 - Stochastics

Random Variables

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Random Variables

Outline

- Definition of a Random Variable
- Probability Distribution Function
    - Important Properties
- Probability Density Function
    - Important Properties
- Common Density Functions

Random Variables

Given a random experiment $\Im$: (S, F, P), a real valued function X(ξ) defined on the probability space is called a random variable. A random variable is subject to the following requirement:

1. For every real number x, the set $\{\xi: X(\xi) \le x\}$ is an event in F.
2. $P(x = \infty) = 0$, $P(x = -\infty) = 0$.

Probability Distribution Function

Probability Distribution Function of a random variable X(ξ) is defines as

$$F_X(x) = P\{X(\xi) \le x\}$$

$$-\infty \le x \le +\infty$$

Probability Distribution Function

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Important Properties

1. $F(-\infty) = 0,\ F(+\infty) = 1$
2. $F(x)$ is non-decreasing
3. $F(x)$ is continuous from the right
4. If $F(x_0) = 0$, $F(x) = 0$ for every $x \le x_0$
5. $P\{X(\xi) > x\} = 1 - F(x)$
6. $P\{x_1 < X \le x_2\} = F(x_2) - F(x_1)$
7. $P\{X = x\} = F(x) - F(x^-)$
8. $P\{x_1 < X \le x_2\} = F(x_2) - F(x_1^-)$

Probability Density Function

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Density Function - Continuous X(ξ)

$$f_X(x) = \frac{dF_X(x)}{dx}$$

Density Function - Discrete X(ξ)

$$f(x) = \sum_i P_i \delta(x - x_i)$$

$$P_i = P\{x = x_i\}$$

Probability Density Function

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Important Properties

1. $f(x) \ge 0$
2. $\int_{-\infty}^{+\infty} f(x)dx = 1$
3. $F(x) = \int_{-\infty}^{x} f(\xi)d\xi$
4. $P\{x_1 < X \le x_2\} = F_X(x_2) - F_X(x_1) = \int_{x_1}^{x_2} f(x)dx$
5. Continuous Random Variable

$$P\{x < X \le x + \Delta x\} \approx f(x)\Delta x$$

$$f(x) = \lim_{\Delta x \to 0} \frac{P(x < X \le x + \Delta x)}{\Delta x}$$

Common Probability Density Function

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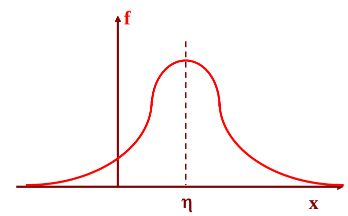
Normal

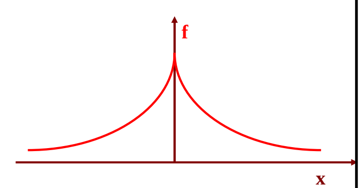
$$f_x(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\eta)^2}{2\sigma^2}}$$

$$F_x(x) = \frac{1}{2} + erf\,\frac{x-\eta}{\sigma}$$



Laplace

$$f(x) = \frac{\alpha}{2} e^{-\alpha|x|}$$



Common Probability Density Function

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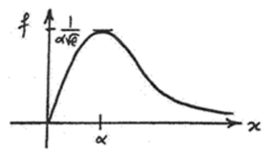
Raleigh

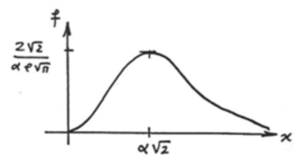
$$f(x)=\frac{x}{\alpha^2}e^{-\frac{x^2}{2\alpha^2}}U(x)$$

$$U(x)=\begin{cases}1 & x\ge 0\\ 0 & x<0\end{cases}$$



Maxwell

$$f(x)=\frac{\sqrt{2}}{\sqrt{\pi}\alpha^3}x^2e^{-\frac{x^2}{2\alpha^2}}$$



Common Probability Density Function

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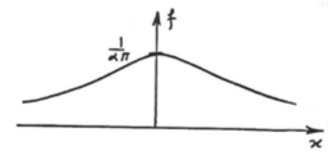
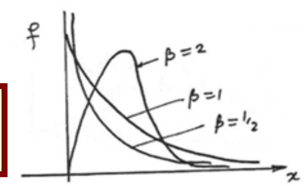
Cauchy

$$f(x)=\frac{\alpha/\pi}{\alpha^2+x^2}$$



Weibull

$$f_x(x)=\begin{cases}kx^{\beta-1}e^{-\alpha x^{\beta}} & x>0\\ 0 & otherwise\end{cases}$$



Common Probability Density Function

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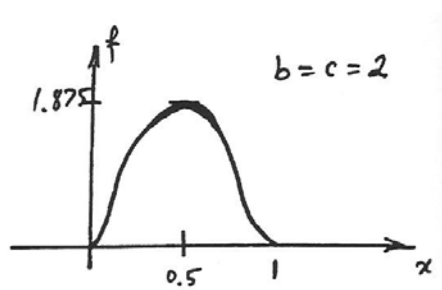
Beta

$$A=\frac{\Gamma(b+c+2)}{\Gamma(b+1)\Gamma(c+1)}$$

$$f(x)=\begin{cases}Ax^b(1-x)^c & 0\le x\le 1\\ 0 & elsewhere\end{cases}$$



Common Probability Density Function

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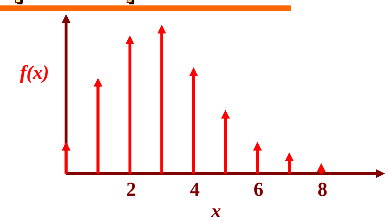
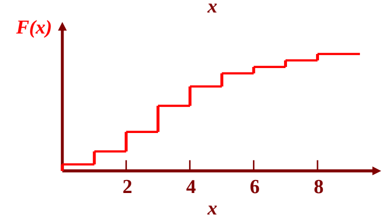
Poisson

$$P\{x(\xi)=k\}=e^{-a}\frac{a^k}{k!}$$

$$f(x)=e^{-a}\sum_{k=0}^{\infty}\frac{a^k}{k!}\delta(x-k)$$





Common Probability Density Function

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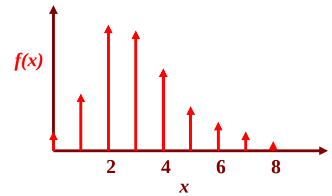
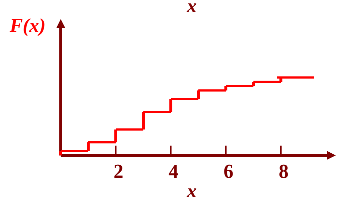
Binomial

$$P\{x=k\}=\binom{n}{k}p^k q^{n-k}$$

$$f(x)=\sum_{k=0}^{n}\binom{n}{k}p^k q^{n-k}\delta(x-k)$$

$$p+q=1$$





Common Probability Density Function

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Erlang

$$f(x)=\frac{c^n}{(n-1)!}x^{n-1}e^{-cx}U(x)$$

Gamma

$$f(x)=\frac{c^{b+1}}{\Gamma(b+1)}x^b e^{-cx}U(x)$$

$$\Gamma(b)=\int_0^\infty y^{b-1}e^{-y}dy$$

Exponential

$$f(x)=ce^{-cx}U(x)$$

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Concluding Remarks

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Thank you!

Questions?