ME 527 **HW SET 1**

1. Which of the following expressions are allowed in index notation? (Problem 3.2)

$a = b_i c_{ij} d_j$, $a = b_i c_i + d_j$, $a_i = \delta_{ij} b_i + c_i$, $a_k = b_i c_{ki}$, $a_k = b_k c + d_i e_{ik}$
$a_i = b_i + c_{ij} d_{ji} + e_i$, $a_l = \varepsilon_{ijk} b_j c_k$, $a_{ij} = b_{ji}$, $a_{ij} = b_i c_j + e_{jk}$, $a_{kl} = b_i c_{ki} d_l + e_{ki}$

2. Prove the following equations using index notation: (Problem 3.6)

$$(\mathbf{a}\times\mathbf{b})\cdot\mathbf{c} = \mathbf{a}\cdot(\mathbf{b}\times\mathbf{c}) = (\mathbf{c}\times\mathbf{a})\cdot\mathbf{b}, \quad \mathbf{t}\times(\mathbf{u}\times\mathbf{v}) = \mathbf{u}(\mathbf{t}\cdot\mathbf{v}) - \mathbf{v}(\mathbf{t}\cdot\mathbf{u}), \quad \mathbf{u}\times\mathbf{v} = -\mathbf{v}\times\mathbf{u}$$

3. Suppose v_i is the velocity at a point in a fluid. Show that $T_{ij} = v_i v_j$ is a tensor.

4. Prove the following equation using indicial notation: (Problem 3.12)

$$div(\phi\mathbf{v}) = \phi div\,\mathbf{v} + \mathbf{v}\cdot grad\,\phi, \quad div(\mathbf{u}\times\mathbf{v}) = \mathbf{v}\cdot\nabla\times\mathbf{u} - \mathbf{u}\cdot\nabla\times\mathbf{v}$$

5. Using indicial notation show that

$$\nabla\times\nabla\times\mathbf{u} = \nabla\nabla\cdot\mathbf{u} - \nabla^2\mathbf{u}.$$

6. Show that

$$\varepsilon_{ijk}\varepsilon_{njk}u_n = 2u_i.$$