ME 527 – Advanced Fluids

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Potential Flow Past a Sphere

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Outline

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- Potential Flow Equation
- Stream Function
- Boundary Conditions
- Stream Function Solution
- Velocity Components

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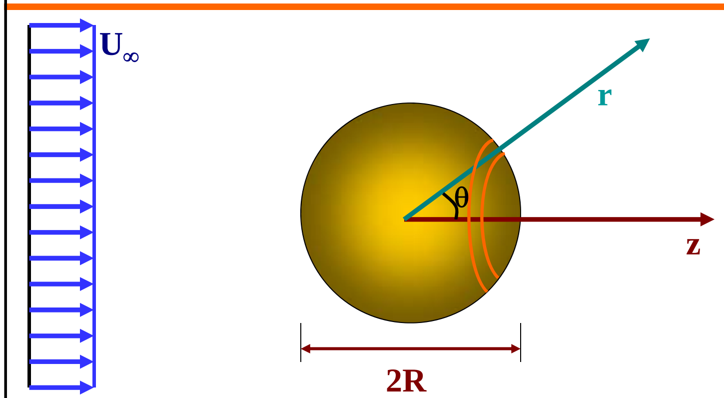

Spherical Coordinates

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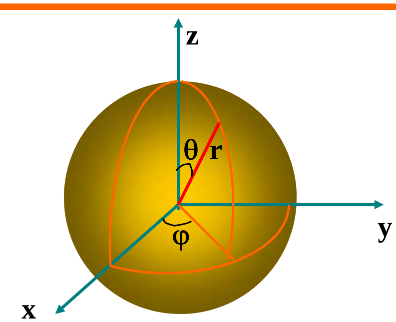
$$\begin{cases} x = r\sin\theta\cos\varphi \\ y = r\sin\theta\sin\varphi \\ z = r\cos\theta \end{cases}$$



Stream Function

$$v_r = \frac{1}{r^2\sin\theta}\frac{\partial\psi}{\partial\theta}$$

$$v_\theta = -\frac{1}{r\sin\theta}\frac{\partial\psi}{\partial r}$$

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Potential Flow Equation

$$E^2\psi = \left[\frac{\partial^2}{\partial r^2} + \frac{\sin\theta}{r^2}\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\right)\right]\psi = 0$$

Boundary Conditions

$$v_r = \frac{1}{r^2\sin\theta}\frac{\partial\psi}{\partial\theta} = 0 \qquad at \;\; r = R$$

$$\psi = \frac{1}{2}U_\infty r^2 \sin^2\theta \qquad as \;\; r \to \infty$$

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Let $\psi = f(r)\sin^2\theta$

N-S $\left(\frac{d^2}{dr^2} - \frac{2}{r^2}\right) f(r) = 0$

Solution $f(r) = Ar^m$

$[m(m-1)-2] = 0$ $\quad m = -1, 2$

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Solution $f(r) = \frac{A}{r} + Br^2$

Boundary Conditions

$$f = 0 \qquad at \;\; r = R$$

$$f = \frac{1}{2}U_\infty r^2 \qquad as \;\; r \to \infty$$

$$A = -BR^3, \quad B = \frac{1}{2}U_\infty, \quad A = -\frac{1}{2}U_\infty R^3$$

$$f(r) = \frac{1}{2}U_\infty\left(r^2 - \frac{R^3}{r}\right)$$

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Solution

Stream Function

$$\psi = \frac{1}{2}U_\infty r^2\left(1 - \frac{R^3}{r^3}\right)\sin^2\theta$$

Velocity Field

$$\frac{v_r}{U_\infty} = \left[1 - \left(\frac{R}{r}\right)^3\right]\cos\theta$$

$$\frac{v_\theta}{U_\infty} = -\left[1 + \left(\frac{R}{r}\right)^3\right]\sin\theta$$

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Streamlines

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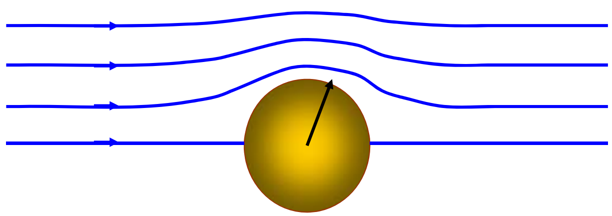
Potential Flow

$$\psi = \frac{1}{2} U_{\infty} r^2 \sin^2\theta \left(1 - \frac{R^3}{r^3}\right)$$



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Moving Sphere-Streamlines

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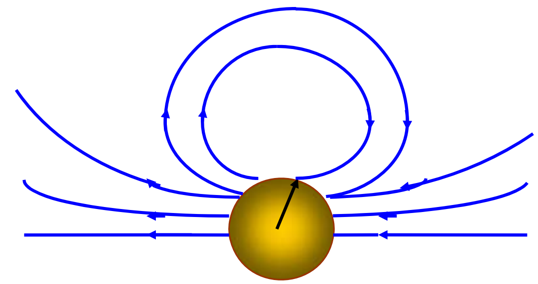
Potential Flow

$$\psi|_{moving} = \psi - \frac{1}{2} U_{\infty} r^2 \sin^2\theta = -\frac{1}{2}\frac{R^3}{r} U_{\infty} \sin^2\theta$$



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Concluding Remarks

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- **Potential Flows**
- **Stream Function**
- **Boundary Conditions**
- **Stream Function Solution**
- **Velocity Components**

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Thank you!

Questions?

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