ME 527 – Advanced Fluids

Clarkson University

FUNDAMENTALS

Goodarz Ahmadi

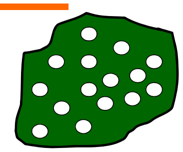
Department of Mechanical and Aeronautical Engineering
Clarkson University
Potsdam, NY 13699-5725

FUNDAMENTALS

Clarkson University

Continuum Assumption

Fluid is a Continuum



3×10^{19} molecules in 1 cm^3

Density:

$$\rho = \lim_{L \to 0} \frac{\sum m_i}{V}$$

Velocity (mass-averaged)

$$\mathbf{v} = \lim_{L \to 0} \frac{\sum m_i \mathbf{v}_i}{\sum m_i}$$

Molar Averaged Velocity

$$\mathbf{V}^{(k)} = \lim_{L \to 0} \frac{\sum \mathbf{v}_i^{(k)}}{n^{(k)}}$$

Properties

Clarkson University

Fluctuation Velocity

$$\mathbf{v}_i' = \mathbf{v}_i - \mathbf{v}$$

Internal Energy Density

$$e = \lim_{L \to 0} \frac{\sum m_i \frac{1}{2} \overline{\mathbf{v}'_i \cdot \mathbf{v}'_i}}{\sum m_i}$$

Temperature

$$\frac{3}{2} kT = \frac{1}{2} m \overline{\mathbf{u}'^2}$$

Thermodynamics

Clarkson University

- Thermodynamic properties (temperature, entropy, internal energy, enthalpy, etc.) are related.
- For a thermodynamic state, all properties are specified.
- A process constitutes a change in state.
- Reversible process is a sequence of thermodynamical state.
- Extensive properties are proportional to the mass of the system.
- Intensive properties are independent of the mass of the system.

Entropy

Entropy measures the irreversibility of a process

$$ds = \frac{dQ}{T}$$

(For reversible processes)

Statistical Mechanics (Boltzmann)

$$s = k \ln f$$

Basic Thermodynamical Equations

$$Tds = de + p d\vartheta$$

$$de = Tds - p d\vartheta = Tds + \frac{p}{\rho^2} d\rho$$

$$T = \left.\frac{\partial e}{\partial s}\right|_{\vartheta}$$

$$p = -\left.\frac{\partial e}{\partial \vartheta}\right|_{s} = \rho^2 \left.\frac{\partial e}{\partial \rho}\right|_{s}$$

Helmholtz Free Energy Function

$$\psi = e - Ts$$

$$\psi = \psi(T, \rho)$$

$$d\psi = de - Tds - sdT = -sdT + \frac{p}{\rho^2} d\rho$$

$$d\psi = \frac{\partial \psi}{\partial T} dT + \frac{\partial \psi}{\partial \rho} d\rho$$

Helmholtz Free Energy Function

$$s = -\left.\frac{\partial \psi}{\partial T}\right|_{p}$$

$$p = \rho^2 \left.\frac{\partial \psi}{\partial \rho}\right|_{T}$$

Enthalpy

$$h = e + \frac{p}{\rho}$$

Isothermal Compressibility

$$\alpha = \frac{1}{\rho} \left.\frac{\partial \rho}{\partial P}\right|_{T}$$

Bulk Expansion

$$\beta = \frac{1}{\rho} \left.\frac{\partial \rho}{\partial T}\right|_{p}$$

Ideal Gas

$$p = \rho RT$$

$$h = e + RT$$

$$c_P = \left.\frac{\partial h}{\partial T}\right|_P = c_V + R$$

$$c_V = \left.\frac{\partial e}{\partial T}\right|_P$$

$$\gamma = \frac{c_P}{c_V}$$

Incompressible Substance

$$c_P = c_V$$

$$\gamma = 1$$

Compressibility Factor

$$Z = \frac{p}{\rho RT}$$

ME 527 G. Ahmadi