ME 527 – Advanced Fluid

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INDICIAL NOTATION

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INDICIAL NOTATION

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Outline

- Basic Rules
- Vectors and Tensors
- Tensor Operation
- Isotropic Tensors

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(Cartesian Tensor)

Basic Rules

- A free index appears only once in each term of a tensor equation. The equation then holds for all possible values of that index.
- Summation is implied on an index, which appears twice.
- No index can appear more than twice in any term.

Definition (Cartesian Tensors)

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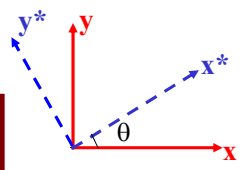
Change of Frame

$$x_i^* = Q_{ij} x_j$$

$$x_j = Q_{ij} x_i^*$$



$$\det|Q_{ij}| = \pm 1$$

$$Q_{ij} Q_{ik} = \delta_{jk} \qquad Q_{ij} Q_{kj} = \delta_{ik}$$

Change of Frame

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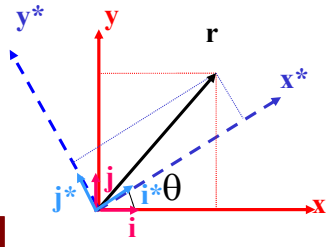
$$\mathbf{r} = x\mathbf{i} + y\mathbf{j} = x^*\mathbf{i}^* + y^*\mathbf{j}^*$$

$$\mathbf{i}^* = \mathbf{i}\cos\theta + \mathbf{j}\sin\theta$$

$$\mathbf{j}^* = -\mathbf{i}\sin\theta + \mathbf{j}\cos\theta$$

$$x^* = x\cos\theta + y\sin\theta$$

$$y^* = -x\sin\theta + y\cos\theta$$



Change of Frame

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Transformation in Two Dimension

$$[Q] = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

Kronecker Delta →

$$[\delta_{ij}] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Definition (Cartesian Tensors)

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Scalar →

$$T^* = T$$

Vector →

$$\mathbf{v}^* = \mathbf{Q}\cdot\mathbf{v}$$

Second Order Tensor

$$\boldsymbol{\tau}^* = \mathbf{Q}\cdot\mathbf{t}\cdot\mathbf{Q}^T$$

Vectors and Tensors

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Vector →

$$v_i^* = Q_{ij}v_j$$

Second Order Tensor →

$$t_{ij}^* = Q_{ik}Q_{jl}t_{kl}$$

Third Order Tensor →

$$\lambda_{ijk}^* = Q_{im}Q_{jn}Q_{kl}\lambda_{mnl}$$

Alternating Symbol

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Alternating Symbol → ε_{ijk}

$$\varepsilon_{ijk} = 1, \quad \text{for i, j, k even permutation}$$
$$\varepsilon_{ijk} = -1, \quad \text{for i, j, k odd permutation}$$
$$\varepsilon_{ijk} = 0, \quad \text{when two indices are equal}$$

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Tensor Operations

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Gradient →
$$(\nabla\varphi)_i = \frac{\partial\varphi}{\partial x_i} = \varphi_{,i}$$
$$(\nabla\mathbf{v})_{ij} = \frac{\partial v_j}{\partial x_i} = v_{j,i}$$

Divergence →
$$\nabla\cdot\mathbf{v} = v_{i,i}$$

$$(\nabla\cdot\boldsymbol{\tau})_j = \frac{\partial\tau_{ij}}{\partial x_i} = \tau_{ij,i}$$

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Curl →
$$(\nabla\times\mathbf{U})_i = \varepsilon_{ijk}\frac{\partial U_k}{\partial x_j} = \varepsilon_{ijk}U_{k,j}$$

Determinant →
$$\det|\mathbf{A}| = \varepsilon_{ijk}A_{1i}A_{2j}A_{3k}$$

Identity →
$$\varepsilon_{ijk}\varepsilon_{imn} = \delta_{jm}\delta_{kn} - \delta_{jn}\delta_{km}$$

Laplacian →
$$\nabla^2\varphi = \frac{\partial^2\varphi}{\partial x_i\partial x_i} = \varphi_{,ii}$$

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Isotropic Tensors

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Rank Zero: → **All Scalars**

Rank One: → **None**

Rank Two: → $\alpha\delta_{ij}$

Rank Three: → $\alpha\varepsilon_{ijk}$

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Isotropic Tensors

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Rank Four:

$$\alpha\delta_{ij}\delta_{kl} + \beta(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}) + \gamma(\delta_{ik}\delta_{jl} - \delta_{il}\delta_{jk})$$

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Concluding Remarks

- Basic Rules
- Vectors and Tensors
- Tensor Operation
- Isotropic Tensors

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Thank you!

Questions?